

Bet error rate performance of channel coding applied on MIMO OSTBC systems

Alsayah Ali M Emhemed¹, Abdullatif M.Omar²

^{1,2} Department of Electrical and Electronic Engineering, College of Technical Sciences, Beni-Walid, Libya

Abstract

This paper explain how to use Orthogonal Space Time Block code (OSTBC) in Multiple Input Multiple Output (MIMO) system with different antenna configurations to improve the performance by maximizing diversity gain. Also investigates the fading and distortion caused by the noise channel. The proposed methodology to reduce the channel effect to get a reliable data transfer is based on encoding the signal at the transmitter then using the error detection and correction algorithms in the receiver to recover the data sent through the channel. To estimate this performance. The bit error rate will be measured and the results will show the optimization in error reduction with different values of signal to noise ratio.

Keywords: orthogonal space time block code (OSTBC), multiple input multiple output (MIMO), alamouti scheme, rayleigh fading channel, AWGN channel, channel coding, bit error rate (BER)

1. Introduction

A Space Time Code is a method employed to improve the reliability of data transmission in wireless communication systems using multiple transmit antennas. From each input symbol from the information source, the space time encoder chooses the constellation point and it simultaneously transmits it from different antennas at different time slots giving coding and diversity gains.

Space-time block coding is a technique used in wireless communications to transmit multiple copies of a data stream across a number of antennas and to exploit the various received versions of the data to improve the reliability of data-transfer. The fact that the transmitted signal must traverse a potentially difficult environment with scattering, reflection, refraction and so on and may then be further corrupted by thermal noise in the receiver means that some of the received copies of the data will be 'better' than others. This redundancy results in a higher chance of being able to use one or more of the received copies to correctly decode the received signal. In fact, space-time coding combines *all* the copies of the received signal in an optimal way to extract as much information from each of them as possible [1].

Channel coding is the technique which improves mobile communication link performance by adding redundant data bits in the transmitted message. In this technique, the base band portion of the transmitter, a channel coder maps a digital message sequence into another specific containing greater number of bits than originally contained in the message. The coded message is then modulated for transmission in the wireless channel. Channel Coding is used by the receiver to detect or correct some or all of the errors introduced by the channel in a particular sequence of message bits. The added coding bits lower these the raw data transmission rate through the channel. There are two types of codes: Block codes and convolutional codes which are introduced and discussed to evaluate of error reduction comparing with the conventional OSTBC MIMO basic system.

2. Orthogonal Space-Time Block Code (OSTBC)

The generalized schemes are referred to as space-time block codes [2]. However, for more than two transmit antennas no

complex valued STBCs with full diversity and full data rate exist. Thus, many different code design methods have been proposed providing either full diversity or full data rate. The concept of STBCs is the provision of full diversity with low complexity. Space-time block codes were designed to achieve the maximum diversity order for the given number of transmit and receive antennas subject to the constraint of having a simple linear decoding algorithm. This has made space-time block codes a very popular and most widely used scheme.

A sequence of P symbols s_1, s_2, \dots, s_p , that are elements of a specific constellation set such as phase shift keying (PSK) or quadrature amplitude modulation (QAM), are mapped into a $T \times N_t$ transmitted matrix X entries of which are the linear combinations of signal This mapping operation is referred to as space-time block encoding. Where T is number channel uses or block length. The symbol rate R is defined as P/T number of symbols per channel use If R equals one, the code defined a full-rate or rate-one code [3].

The STBC mapping of symbols $[s_1, s_2, s_3, \dots, s_N]$ is arranging these symbols in a matrix S of dimension $nt \times N$:

$$S = \begin{bmatrix} s_1^1 & s_2^1 & \dots & s_N^1 \\ s_1^2 & s_2^2 & \dots & s_N^2 \\ \vdots & \vdots & \ddots & \vdots \\ s_1^{n_t} & s_2^{n_t} & \dots & s_N^{n_t} \end{bmatrix}$$

The i^{th} row $[s_1^i, s_2^i, s_3^i, \dots, s_N^i]$ is the data sequence transmitted from the i^{th} transmit antenna and the j^{th} column $[s_j^1, s_j^2, s_j^3, \dots, s_j^{n_t}]$ is the space-time symbol transmitted at time j , $1 \leq j \leq N$.

The STBC encoded streams are sent to the channel through the transmit antennas complex number h which is called the fade co-efficient arranged channel H matrix [4]:

$$H = \begin{bmatrix} h_{1,1} & h_{1,2} & \dots & h_{1,n_t} \\ h_{2,1} & h_{2,2} & \dots & h_{1,n_t} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_r,1} & h_{n_r,2} & \dots & h_{n_r,n_t} \end{bmatrix}$$

Assuming r is the received signals matrix therefore it can be written matrix form:

$$r = S.H + n$$

Where n is the additive Gaussian noise signals matrix at the receiver antenna. Assuming fade coefficients h_{ij} ($0 < i \leq N_r$) and ($0 < j \leq N_t$) and for each corresponding path between transmitter j and receiver i , we have noise at each receiver N_1, \dots, N_r . Then the equation of i^{th} receiver signal as:

$$r_i = s_{i1}h_{i1} + s_{i2}h_{i2} + \dots + s_{iN_t}h_{iN_t} + n_i$$

The orthogonal space-time block code for any number of transmits antennas nt at N transmission period, is described by a $nt \times N$ transmission matrix S , The code matrix S in OSTBC should satisfies the orthonormality property :

$$S^H S = \sum_{n=1}^N |S_n|^2 I$$

S^H represents the Hermitian (conjugate) transpose of, and I identity matrix.

2.1 Alamouti's Scheme in MIMO 2x2 Configuration

There Alamouti scheme is the basis of the Space Time Coding technique, that provides full diversity at full data rate The scheme uses two transmit antennas and M Receive antennas to provide a diversity order of $2M$. In every two time intervals two symbols are transmitted. The symbols are produced from the digital modulation of data source bits then Alamouti space-time Encoder takes the two modulated symbols to create encoding matrix S where the symbols s_1 and s_2 are mapped to two transmit antennas in two transmit time slots. The encoding matrix is given by [5]:

$$S = \begin{bmatrix} S_1 & S_2 \\ -S_2^* & S_1^* \end{bmatrix}$$

Assuming at the receiver 2 receive antennas receives r_1 and r_2 denoting the two received signals over the two consecutive symbol periods for time t and $t+T$. The scheme then follows the same pattern for all subsequent symbols, so that received signal can be represented by the equations

$$\begin{aligned} r_1(t) &= h_{11}s_1 + h_{21}s_2 + n_1(t) \\ r_1(t+T) &= -h_{11}s_2^* + h_{21}s_1^* + n_1(t+T) \\ r_2(t) &= h_{12}s_1 + h_{22}s_2 + n_2(t) \\ r_2(t+T) &= -h_{12}s_1^* + h_{22}s_2^* + n_2(t+T) \end{aligned}$$

The table 1 describes the sending structure of these samples:

Table 1: Signals encoding by Alamouti's Scheme

	Antenna 1	Antenna 2
Time t	s_1	s_2
Time t+T	$-s_2^*$	s_1^*

The decoder builds the following two signals that are sent to maximum likelihood detector:

$$\begin{aligned} \tilde{s}_1 &= h_1^* r_1 + h_2^* r_2 \\ \tilde{s}_2 &= h_2^* r_1 - h_1^* r_2 \end{aligned}$$

Then at the receiver the Alamouti decoder recovers the transmitted data, by estimating the channel coefficients and combining received signals.

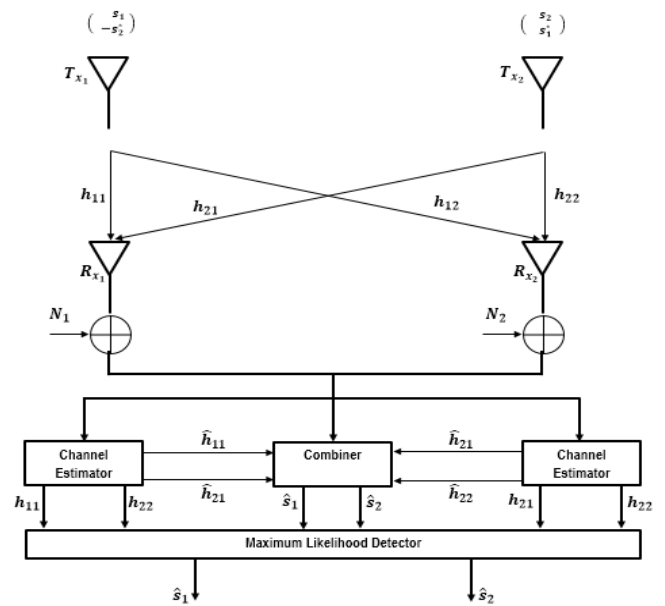


Fig 1: MIMO 2x2 System using Alamouti's Scheme

2.2 OSTBC in MIMO 3x3 Configuration

It The MIMO 3x3 scheme uses 3 antennas on the transmitter and 3 on the receiver, in this case there is an additional parameter which produce different structures of OSTBC Code word Matrix, this parameter is denoted as code rate which is defined as a ratio between number of transmit antenna to the number of time periods the channel and the received. Here the chosen code rate is $3/4$, thus the three symbols transmitted in four time slots according to the following encoding matrix [6]:

$$S = \begin{bmatrix} s_1 & s_2 & s_3 \\ -s_2^* & s_1^* & 0 \\ s_3^* & 0 & -s_1^* \\ 0 & s_3^* & -s_2^* \end{bmatrix}$$

The OSTBC encoder codes each 3 consecutive complex samples at time t_0 : the samples s_1, s_2 and s_3 are respectively sent through the antenna number 1, 2 and 3. For four consecutive times, for all time periods TABLE 2 explains the distribution of samples on the antennas during the time

Table 2: Signals encoding by 3x3 Configuration

	Antenna 1	Antenna 2	Antenna 3
Time t	s_1	s_2	s_3
Time t+T	$-s_2^*$	s_1^*	0
Time t+2T	s_3^*	0	$-s_1^*$
Time t+3T	0	s_3^*	$-s_2^*$

According to the table the received signals from each i^{th} transmit antenna can be represented of each by the equations

$$\begin{aligned} r_i(t) &= h_{1i}s_1 + h_{2i}s_2 + h_{3i}s_3 + n_i(t) \\ r_i(t+T) &= -h_{1i}s_2^* + h_{2i}s_1^* + n_i(t+T) \\ r_i(t+2T) &= h_{1i}s_3^* - h_{3i}s_1^* + n_i(t+2T) \\ r_i(t+3T) &= h_{2i}s_3^* - h_{3i}s_2^* + n_i(t+2T) \end{aligned}$$

The OSTBC Combiner block recovers the transmitted data, by estimating the channel coefficients and combining received signals according to the following equation.

$$\begin{pmatrix} \hat{s}_1 \\ \hat{s}_2 \\ \hat{s}_3 \end{pmatrix} = \frac{1}{\|H\|^2} \sum_{j=1}^M \begin{pmatrix} h_{1,j}^* r_{1,j} + h_{2,j} r_{2,j}^* - h_{3,j} r_{3,j}^* \\ h_{2,j}^* r_{1,j} - h_{1,j} r_{2,j}^* - h_{3,j} r_{4,j}^* \\ h_{3,j}^* r_{1,j} + h_{1,j} r_{3,j}^* + h_{2,j} r_{4,j}^* \end{pmatrix}$$

\hat{s}^k represents the estimated k th symbol in the OSTBC code word matrix. h_{ij} represents the estimate for the channel from the i th transmit antenna and the j th receive antenna. The values where M is the number of receive antennas and

$$\|H\|^2 = \sum_{i=1}^N \sum_{j=1}^M \|h_{ij}\|^2$$

Where N is the number of transmit antennas.

3. Channel Coding

The Reduction of error due to noisy channels becomes main challenge to get a reliable communication system, there for channel coding is suggested to enhance the data transfer in high rates through multipath fading channel. Channel coding concept is mainly error detection and correction it and helps the communication systems design to reduce the noise effect during transmission.

Channel codes can be stratified broadly in two categories: block codes and convolution codes. For real-time error correction, convolution codes are preferred. Smaller code word usage in convolution codes can achieve the same quality as obtained with longer code-words in block coding. In block codes, n bit code-word can be obtained by encoding a block of k information bits. Whereas in convolution coding, encoded n bit sequence depends on previous information bits too along with current information bits [7].

Examples of block codes are Hamming codes, Reed-Solomon codes and Low-Density Parity-Check codes (LDPC). A code having a minimum distance d_{min} is capable of correcting all patterns of errors $t = (d_{min} - 1)/2$ or fewer errors in a code word, and is referred to as a random error correcting code.

Bit interleaving is a well-known technique for dispersing the errors that occur in burst when the received signal level fades, and which are likely to exceed the error correcting capability of a code [8]. Before a message is transmitted, the entire bit stream is interleaved. Hence the burst errors will be shared among the interleaved code words and only a simple code is required to correct them. Note that the interleaving process does not involve adding redundancy. Concatenated coding schemes are used to provide even more protection against bit errors than is possible with a single coding scheme.

3.1 Reed-Solomon Codes

Reed-Solomon codes are *non-binary* cyclic codes with symbols made up of m -bit sequences, where m is any positive integer having a value greater than 2. R-S (n, k) codes on m -bit symbols exist for all n and k for which $0 < k < n < 2^m + 2$ where k is the number of data symbols being encoded, and n is the total number of code symbols in the encoded block. For the most conventional R-S (n, k) code,

$$(n, k) = (2m - 1, 2m - 1 - 2t)$$

Where t is the symbol-error correcting capability of the code, and $n - k = 2t$ is the number of parity symbols.

Reed-Solomon codes achieve the largest possible code minimum distance for any linear code with the same encoder input and output block lengths. For non-binary codes, the distance between two code words is defined as the number of symbols in which the sequences differ. For Reed-Solomon codes, the code minimum distance is given by [9]

$$d_{min} = n - k + 1$$

3.2 Convolutional Code

In telecommunication, convolutional code is a type of error-correcting code in which each m -bit information symbol (each m -bit stream) to be encoded is transformed into an n -bit symbol, where m/n is the code rate ($n > m$). The transformation is a function of the k information symbols [10]. To convolutional encoder data, start with a k memory registers, each holding one bit input, All memory registers start with a value of zero, unless otherwise specified, The encoder has n adders are implemented with a single Boolean XOR gate, there are two basic components of the convolutional encoder (flip-flops including the shift register and exclusive- or gates comprising the associated modulo-two adders) defined, Fig.2 shows a convolutional encoder for a rate $1/2$, $K = 3$, $m = 2$.

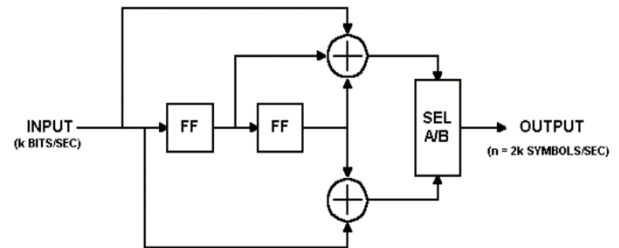


Fig 2: Example of Convolutional Encoder

In this encoder, data bits are provided at a rate of k bits per second. Channel symbols are output at a rate of $n = 2k$ symbols per second. The input bit is stable during the encoder cycle. The encoder cycle starts when an input clock edge occurs. When the input clock edge occurs, the output of the left-hand flip-flop is clocked into the right-hand flip-flop, the previous input bit is clocked into the left-hand flip-flop, and a new input bit becomes available. Then the outputs of the upper and lower modulo-two adders become stable. The output selector cycles through two states-in the first state, it selects and outputs the output of the upper modulo-two adder; in the second state, it selects and outputs the output of the lower modulo-two adder. The encoder shown above encodes the $K = 3$, $(7, 5)$ convolutional code. The octal numbers 7 and 5 represent the code generator polynomials, which when read in binary (111_2 and 101_2) correspond to the shift register connections to the upper and lower modulo-two adders, respectively. This code has been determined to be the "best" code for rate $1/2$, $K = 3$. It is the code I will use for the remaining discussion and examples, for reasons that will become readily obvious when we get into the Viterbi decoder algorithm [11].

The significant concept to in understanding the Viterbi algorithm is the trellis diagram. The figure below shows the trellis diagram for our example rate $1/2$ $K = 3$ convolutional encoder, for a 15-bit message:

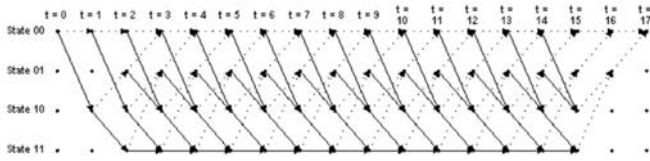


Fig 3: Trellis Diagram

Each time of receiving a pair of channel symbols, the "distance" between is measured all of the possible channel symbol pairs could be received. The measured distance is computed by simply counting how many bits are different between the received channel symbol pair and the possible channel symbol pairs. The results can only be zero, one, or two. The dotted lines represent cases where the encoder input is a zero, and solid lines represent cases where the encoder input is a one. The paths through the trellis corresponding to the actual message, shown in bold, is still associated with the smallest accumulated error metric. This is the thing that the Viterbi decoder exploits to recover the original message. Fig.4 explains the operations from $t=1$ to $t=5$ and Fig.5 shows the final step of decoding.

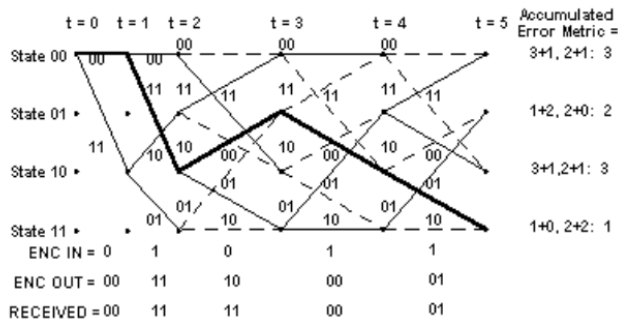


Fig 4: Viterbi Decoding at $t=5$

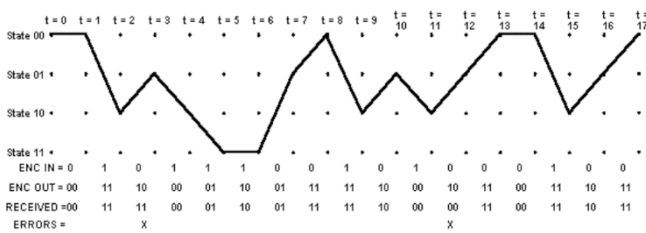


Fig 5: Viterbi Decoding Final step

3.3 Interleaving

Interleaving is a process that makes a system more efficient, fast and reliable by arranging data in a non-contiguous manner. Uses of interleaving, the simplest form of interleaver is the Rectangular Block Interleaver which works by writing the input data symbols into a rectangular memory array in a certain order and then reading them out in a different, mixed-up order. The input symbols must be grouped into blocks. Unlike the Convolutional Interleaver, where symbols can be continuously input, the Rectangular Block Interleaver inputs one block of symbols and then outputs that same block with the symbols rearranged.

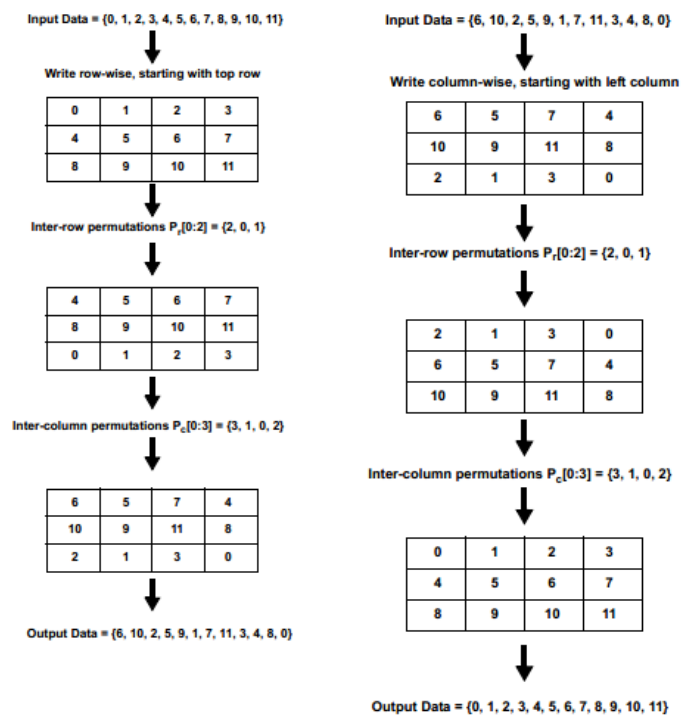
The Rectangular Block Interleaver operates as follows:

1. All the input symbols in an entire block are written row-wise, left to right, starting with the top row.
2. Inter-row permutations are performed if required.
3. Inter-column permutations are performed if required.
4. The entire block is read column-wise, top to bottom, starting with the left column.

The de-interleaver operates in the reverse way:

1. All the input symbols in an entire block are written column-wise, top to bottom, starting with the left column.
2. Inter-row permutations are performed if required.
3. Inter-column permutations are performed if required.
4. The entire block is read row-wise, left to right, starting with the top row [12].

An example of Rectangular Block Interleaver operation is shown in Fig. 6



Block Interleaving

Block De-interleaving

Fig 6: Trellis Diagram

4. The experiment work

The study work will describe a MATLAB/SIMULINK simulation of the MIMO OSTBC system. The study will presents with the system performance by calculating the Bit Error Rate with different values of the Signal to Noise Ratio. The aim of this work is to deeply investigate the behavior of an MIMO Rayleigh fading channel. Then this system can be optimized using channel coding technique to minimize the Bit Error Rate, the simulation have two cases based on the antenna space diversity configurations, first uses the Alamouti scheme of 2×2 system, and the other uses a 3×3 system with code rate $3/4$. The MIMO OSTBC system in this study contains two subsystems having same data source, first subsystem is a conventional OSTBC system consists of two blocks, the 64-QAM modulator and OSTBC encoder at the transmitter and

two equivalent blocks, the 64-QAM demodulator and OSTBC combiner the receiver, the second subsystem consists similar blocks of the first subsystem with added blocks represent the channel coding algorithm, the channel coding blocks are Reed Solomon Encoder, Convolutional Encoder, and the Interleaver at the transmitter, and Reed Solomon Decoder, Convolutional Decoder, and the Deinterleaver at the receiver. The transmitter and the receiver are connected through a Rayleigh fading channel the system block diagram is built as in Fig.7.

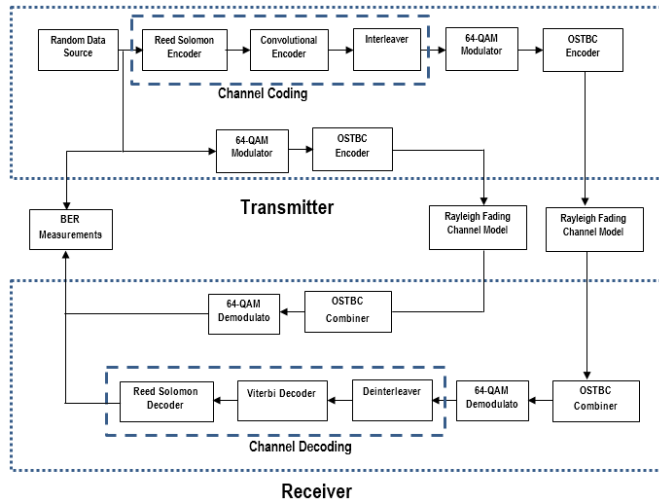


Fig 7: The MIMO OSTBC system block diagram

Then Simulink model which represents the system as explained in previous figure is built by creating a new model file, adding the related blocks from the library, and connecting them as in Fig.8.

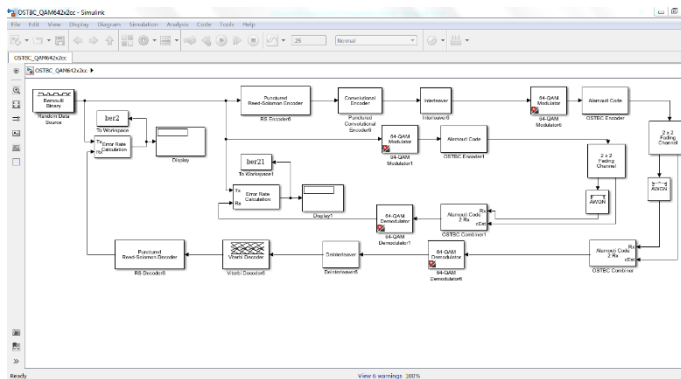


Fig 8: The system setup in Simulink Environment

The rate calculation blocks are connected to the output of the binary data source and the output terminals at the receiver of both subsystems used to compute the error rate of the received data by comparing it to the input data. The block output is a three-element vector consisting of the error rate, followed by the number of errors detected and the total number of symbols compared.

This experiment have two cases which consider the performance according to the antenna configuration the adjustment of the configuration is mainly implemented by modelling the MIMO channel and changing the settings of the OSTBC Encoder and Combiner, the modeling of 2x2 and 3x3 is constructed as in Fig.'s 9 and 10 respectively.

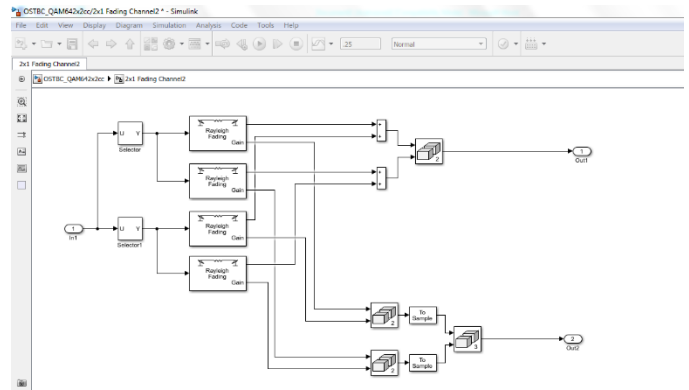


Fig 9: The 2x2 MIMO channel modeling

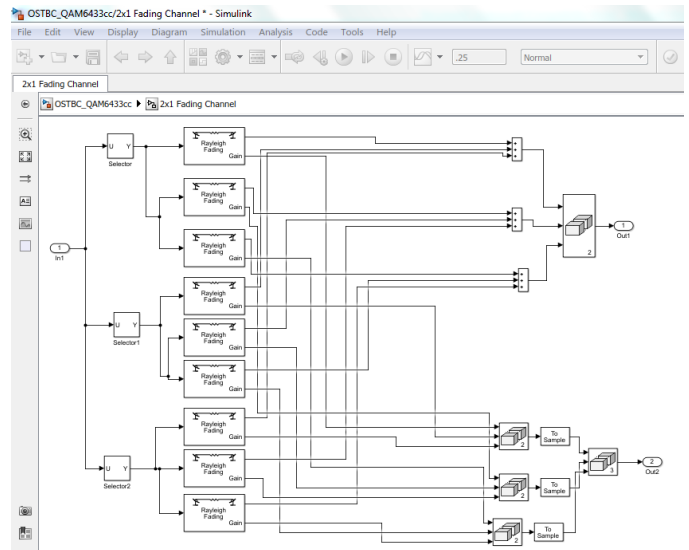


Fig 10: The 3x3 MIMO channel modeling

by defining the SNR values in the AWGN channel Block then run the Simulink model and be repeating this operation as number of different SNR value the corresponding BER will be calculated at each time to be plotted using MATLAB. In every run time the Bit Error rate will be restored and arranged in the data set with the correspondent SNR data vector. The results of BER according to the SNR values from the two 2x2, and 3x3 systems are shown in Table 3

Table 3: The BER performance of 2x2 and 3x3 OSTBC

SNR	2x2 with Channel coding	2x2	3x3 with channel coding	3x3
0	0.2514	0.4996	0.1577	0.4903
5	0.1439	0.4899	0.0688	0.3738
10	0.0578	0.3673	0.0140	0.0776
11	0.0440	0.2990	0.0085	0.0359
12	0.0316	0.2147	0.0047	0.0129
13	0.0213	0.1265	0.0024	0.0027
14	0.0133	0.0576	0.0010	0.0001
15	0.0075	0.0179	0.0004	4.67x10 ⁻⁶
16	0.0038	0.0033	0.0001	0
17	0.0017	0.0002	2.57x10 ⁻⁵	0
18	0.0006	8.66x10 ⁻⁶	5.33x10 ⁻⁶	0
20	3.77x10 ⁻⁵	0	0	0
22	1.54x10 ⁻⁶	0	0	0
25	0	0	0	0
28	0	0	0	0
30	0	0	0	0

The BER results obtained using the simulation is presented in Fig.11 which shows the performance of 2x2, and 3x3 MIMO OSTBC in term of channel coding.

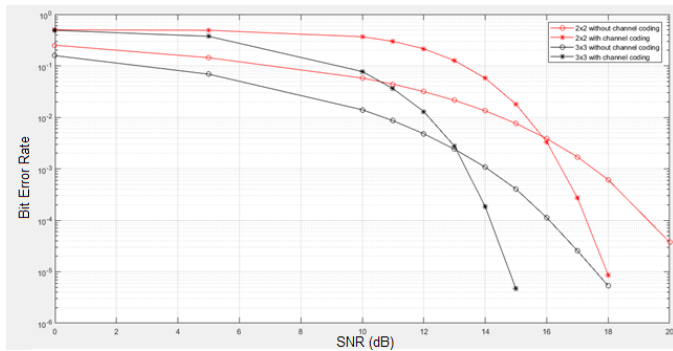


Fig 11: The BER calculations with related Channel SNR

The results obtained using the simulation methodology is shown in Fig. 11. This Figure shows the BER performances of 2x2, and 3x3 MIMO–OSTBC systems with two cases of each system explaining the performance of channel coding technique. It is clearly observed, the overall performance of the 3x3 system is much better than 2x2 system. For 2x2 system the BER is not improved in case of channel coding until SNR = 16 dB then the BER is reduced much rapidly and get closer to zero. Also for 3x3 system the BER reduction is significantly improved in case of channel coding at SNR > 13 dB to be zero at SNR = 15 while in case of channel coding not included the BER reach to zero at SNR = 18.

5. Conclusion

In the In this paper a MIMO OSTBC system has been introduced with different antenna spacing diversity, the selected antenna configuration are 2x2 Alamouti scheme and 3x3 code rate ¾ which also explored in this paper. The improvement criteria using channel coding is purposed to optimize the system in terms of error reduction.

From the simulation results shown in tables and plots, it is found that error reduction is get better performance as the while as the number of antennas increases. Also the channel coding performance is significantly observed with high SNR values where the BER decreases rapidly comparing to the conventional OSTBC system.

6. References

1. Clerk Maxwell J. A Treatise on Electricity and Magnetism, 3rd ed., Oxford: Clarendon. 1892; 2:68-73.
2. Jafarkhani H. Space-Time Coding Theory and Practice, in Magnetism, Cambridge University Press. UK.
3. Erotokritou ID. Space-Time Block Coding For Multiple Transmit Antennas Over Time-Selective Fading Channels, Louisiana State University, 2004, USA.
4. Gupta S, Dutta R, Tiwari AC. Performance Analysis of Alamouti and Orthogonal Space -Time Block Codes In MIMO System under Rayleigh Fading Scenario”, International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering. 2014; 3(10).
5. Al-Dwairi M, Oraiqat M, AlQadi Z, Bdair M. Evaluation of Alamouti Space-time Coding with Virtual Antenna Arrays. World Applied Sciences Journal. 2010; 11(2):127-131.

6. Patel KT. Performance of Stbc, Spatial Multiplexing and Hybrid Technique For Mimo Odfm System. International Journal of Software Engineering Research & Practices. 2016; 2(3).
7. Parastoo S, Predrag R. On Information Rate of Time-varying Fading Channels Modeled as Finite-State Markov Channels, IEEE Transactions on Communication, 56(8).
8. Hyunseung C. Mobile Radio Propagation/Channel Coding Mobile Computing. Sungkyunkwan University March 11, 2011.
9. Wicker S, Bhargava V. Reed-Solomon Codes and Their Applications (Piscataway, NJ: IEEE Press, 1983.
10. Gunduz D, Erkip E. Joint Source-Channel Codes for MIMO Block-Fading Channels, IEEE Transactions on Information Theory. 2008; 54(1):116-134.
11. Haene S, Burg A, Perels D, Luethi P, Felber N, Fichtner W. FPGA Implementation of Viterbi Decoders for MIMO-BICM, in Conference Record of the Thirty-Ninth Asilomar Conference on Signals, Systems and Computers, 2005.
12. Iqbal Z, Nooshabadi S. Effects of Channel Coding and Interleaving in MIMO-OFDM Systems, in 54th IEEE International Midwest Symposium on Circuits and Systems (MWSCAS), Seoul, South Korea, 2011.