



Optimization of Matching between Mechanics and Thermodynamics for Engine Efficiency Improvement

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Abstract

The relationship between engine mechanics and thermodynamics is investigated in this paper. By means of numerical simulation, the inherent mismatching between the mechanics behaviors and the thermodynamic process in internal combustion engines is identified, which is believed to be one of the important limiting factors of energy efficiency for the conventional slider crank engines available in the current market. A possible novel approach is proposed and discussed for engine efficiency improvement through Optimization of Matching between Engine Mechanics and Thermodynamics (OMBMT). A parameter of Matching Gain is defined for quantifying engine efficiency improvement by comparing with a baseline engine. Several case studies have been conducted toward the actual designs in the history of engine development. The reasons for positive gains achieved as well as for negative results obtained are interpreted with the matching concept. Using this matching concept as a guideline, discussions on possible directions for engine efficiency improvement are made based on the findings identified with this method presented. Implementing the principles proposed in this paper, a novel mechanism is being proposed and a concept engine is being modeled and developed for matching among 4 individual strokes in a 4-stroke engine.

Keywords: engine, optimization, matching between mechanics and thermodynamics, matching gain, engine efficiency improvement, coincidence

1. Introduction

Internal combustion engine (ICE) is our prime mobile power source to drive vehicles today. Modern gasoline engine has 30% to 40% fuel conversion efficiency. This means that only 30% to 40% of the energy in the consumed fuel is converted into mechanical power, while the rest is lost through friction and heat. Due to millions of ICEs currently in use worldwide, the improvement in ICE fuel conversion efficiency will have huge impacts on energy consumption, fuel economy, fuel reserve and the environment.

To explore the possibilities for ICE energy efficiency improvements, many studies have contributed the advanced methodologies and strategies. Over 60 engine research projects under the Advanced Combustion Engine Program, led and funded by the U.S. Department of Energy are collected and summarized by Singh, G. *et al.* [1]. There have been also analysis and proposals [2] toward engine efficiency improvements. It appears that the majority of engine efficiency improvement studies are concentrated mostly on engine combustion and thermodynamics. More sophisticated models are built for understanding and improving the detailed combustion processes within the existing frame of engine mechanism. On the other hand, many engine mechanics studies have been focused on vibration characteristics aiming to improve engine lifespan, noise and vibration reduction, etc. [3, 4, 5, 6]. Discussions on the trade-off between mechanics and thermodynamics are often in association with designs of specific engines. A study was conducted by Opaliniski *et al.* [7] on relationship between engine efficiency and various engine configurations, however, engine mechanical features have been neutralized by math-averaging, and their "eigenvalue" contributions to engine efficiency have not been identified.

It is well known that ICE performance depends on the interaction of engine mechanics and thermodynamic processes. Thermal energy from combustion must be applied to engine mechanics to be converted into mechanical energy. How engine mechanics behave under thermodynamics process to capture energy from combustion affects engine efficiency.

To gain deeper insights into the relationship between engine mechanics and thermodynamics and to find leverage points and to explore more possible ways for engine efficiency improvement, a study on slider crank engines has been conducted and some results are presented briefly in this paper.

2. Engine Mechanics Model

Engine mechanics models can be found from many references such as [3, 4, 5, 6]. Usually they are presented as functions of crank angle θ . For a conventional crank-based reciprocal engine, the major parameters used in engine mechanics models are piston position $X(\theta)$ from piston pin to TC, piston position from crank center to piston pin center $x_p(\theta)$, piston speed $s_p(\theta)$, maximum piston speed S_{pmax} and crank angle of the maximum piston speed θ_{spmax} .

$$X(\theta) = (1 + r) - x_p(\theta) \quad (1)$$

$$x_p(\theta) = r \cdot \cos(\theta) \pm \sqrt{l^2 - [r \cdot \sin(\theta)]^2} \tag{1a}$$

$$s_p(\theta) = r \cdot \sin(\theta) \cdot [1 + \cos(\theta) \cdot \frac{r}{l}] \tag{2}$$

where r is crank radius, l is connecting rod length. Let $L = l/r$ which is referred as relative connecting rod length, and let $r = 1$, then the normalized piston speed $s_{p,n}(\theta)$ is

$$s_{p,n}(\theta) = \sin(\theta) \cdot [1 + \frac{\cos(\theta)}{L}] \tag{3}$$

Fig.1 shows the curves of $s_{p,n}(\theta)$ over L , in the typical values of 1.40 and 3.40, which reflects the influence of engine configurations (connecting rod length over crankshaft radius ratio $L = l/r$) on the piston speed. When L tends to infinity, the related $s_{p,n}(\theta)$ curve becomes a sine wave, which is also shown in Fig.1 to present an important engine structure.

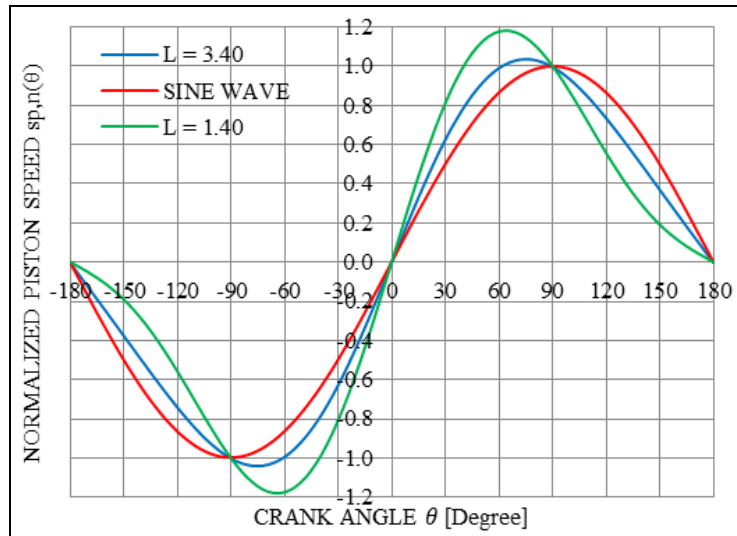


Fig 1: Curves of Normalized Piston Speed $s_{p,n}(\theta)$ vs $L = l/r$ Ratio

As a special case, when $\theta = \pm 90^\circ$, $\sin(\theta) = \pm 1$, $\cos(\theta) = 0$,

$$s_{p,n}(\theta = \pm 90^\circ) = \pm 1.00 \tag{4}$$

Eq.4 presents the golden point for normalization where all the curves in the family of $s_{p,n}(\theta)$ meet. That is why every curve goes across points of $(-90^\circ, -1.00)$, $(90^\circ, 1.00)$ in Fig.1.

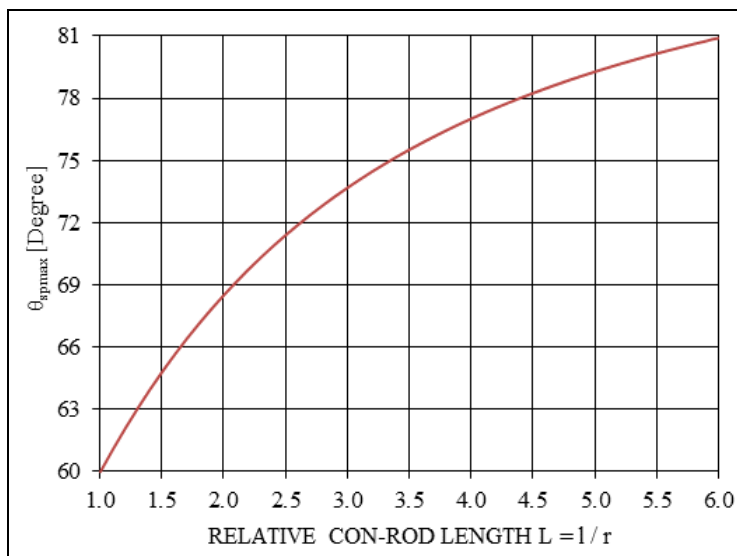


Fig 2: Crank Angle of the Maximum Piston Speed θ_{spmax}

Next, crank angle position of the maximum piston speed θ_{spmax} will be

$$\theta_{spmax} = \cos^{-1} \left[\left(\frac{L}{4} \right) \cdot \left(\sqrt{\frac{8}{L^2} + 1} - 1 \right) \right] \tag{5}$$

From Eq.5, we may find out when L tends to infinity, the term inside the square brackets becomes zero, thus θ_{spmax} tends to 90° . Another extreme case is when L tends to unity, the term inside the square brackets becomes 0.50, thus θ_{spmax} tends to 60° . Fig.2 shows the curve of θ_{spmax} over the L range of 1.00 to 6.00. It can be seen that the longer the relative connection rod, the later the crank angle position of the maximum piston speed θ_{spmax} occurs from TC at $\theta = 0^\circ$.

Utilizing θ_{spmax} derived from Eq.5, the normalized maximum piston speed $S_{pmax,n}$ can be calculated by

$$S_{pmax,n} = S_{p,n}(\theta_{spmax}) = \sin(\theta_{spmax}) \cdot \left[1 + \frac{\cos(\theta_{spmax})}{L} \right] \tag{6}$$

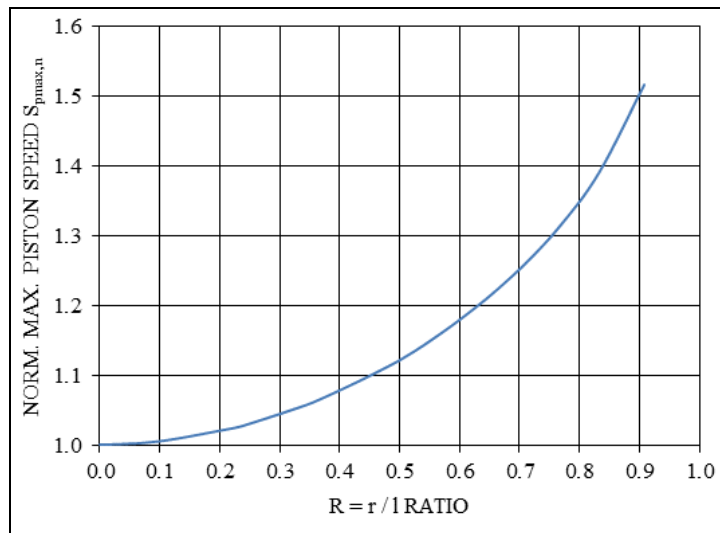


Figure 3. Normalized Maximum Piston Speed $S_{pmax,n}$

It can be seen from Fig.3, the shorter the relative connection rod, the larger the $R = l / L$, the higher the normalized maximum piston speed $S_{pmax,n}$.

To cover the range of L from 1.00 to infinity, $S_{pmax,n}$ and θ_{spmax} could be calculated and presented by using R , the reciprocal of L in Fig.3 and Fig.4 respectively, where R is in range of (0, 1) and $R = 1 / L = r / l$.

3. Engine Thermodynamics Model

In engine thermodynamics, the most frequently used model is the work done within the power stroke [3]:

$$W = \int P dV = \int cV^{-\gamma} dV = \frac{\Delta(PV)}{1-\gamma} \tag{7}$$

where γ presents gas specific heat ratio and $\Delta(PV)$ denotes the work change which must be based on volume change, and also related to compression ratio. Compression ratio and the gas specific heat ratio γ are thermodynamics related parameters.

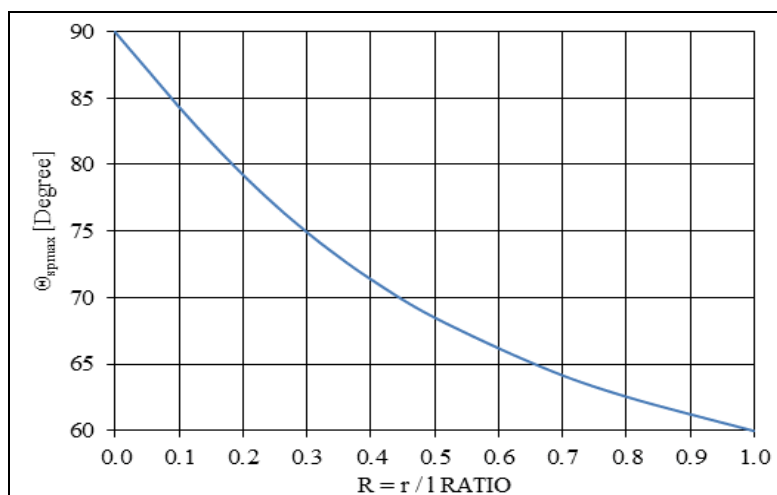


Fig 4: Crank Angle of the Maximum Piston Speed θ_{spmax}

Cylinder gas pressure $p_g(\theta)$ is one of the most important variables related to thermodynamics model. It connects the combustion process to engine mechanism and exerts the entire engine mechanism to operate. It is an instantaneous function of crank angle θ . Different engine mechanism will have different piston travel and different cylinder volume history, causing different shapes of $p_g(\theta)$ curves. In the following case studies, cylinder gas pressure curves $p_g(\theta)$ for the various engine design cases are derived from the cylinder volume history according to ideal gas law under the conditions of identical initial pressure, initial volume, compression ratio, gas specific heat ratio γ and heat release rate.

Fig.5 shows the curves of the $p_g(\theta)$ models used for case comparison studies, in which the upper curve belongs to sine wave engine; the middle one belongs to conventional crank-based engine with $L = 3.40$ and the lower one belongs to short connecting-rod engine with $L = 1.40$. All these curves are derived from the $s_{p,n}(\theta)$ curves in Fig.1, with the initial pressure of 100 kPa, compression ratio of 8.00 and $\gamma = 1.35$, and also the identical heat addition. The heat release rate involved follows the Wiebe model [3], with Wiebe efficiency factor $n = 3$; Wiebe form factor $a = 5$; starting combustion crank angle = -21° and combustion duration angle = 42° .

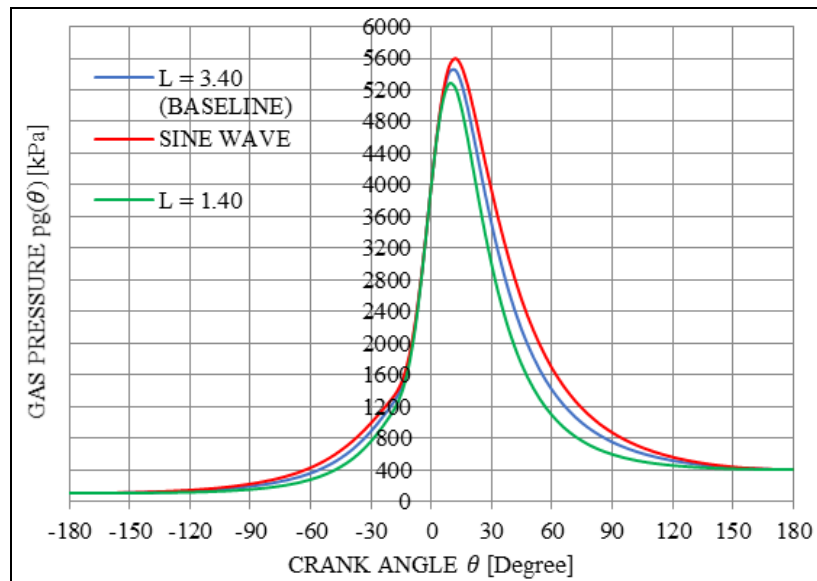


Fig 5: Cylinder Gas Pressure $p_g(\theta)$, Compression Ratio = 8.00, $\gamma = 1.35$

4. Relationship between Engine Mechanics and Engine Thermodynamics

The work done over 2-cylinder enclosed strokes within one engine cycle W_{ps} can be expressed as follows:

$$\begin{aligned}
 W_{ps} &= \oint P dV = \int_{BC1}^{TC} p_g(\theta) \cdot v(\theta) d\theta + \int_{TC}^{BC2} p_g(\theta) \cdot v(\theta) d\theta \\
 &= \int_{BC1}^{TC} p_g(\theta) \cdot A_p \cdot s_p(\theta) d\theta + \int_{TC}^{BC2} p_g(\theta) \cdot A_p \cdot s_p(\theta) d\theta
 \end{aligned}
 \tag{8}$$

where A_p is piston area, TC is the top center and BC is the bottom center of the piston stroke ($BC1$ denotes BC before TC ; $BC2$ presents BC after TC). The integration in Eq.8 covers the full range of compression stroke and power stroke. Eq.8 also contains 2 functions of crank angle θ , $p_g(\theta)$ and $s_p(\theta)$. Obviously, $p_g(\theta)$ belongs to thermodynamics domain while $s_p(\theta)$ belongs to mechanics domain. Both interact with each other to generate torque for driving the crankshaft. The relationship between engine mechanics and engine thermodynamics becomes natural and indivisible for engine work output and efficiency. Let $A_p = 1$, $r = 1$, then the normalized work done within compression and power strokes in one engine cycle, $W_{cps,n}$ becomes

$$\begin{aligned}
 W_{cps,n} &= \int_{BC1}^{TC} p_g(\theta) \cdot s_{p,n}(\theta) d\theta + \int_{TC}^{BC2} p_g(\theta) \cdot s_{p,n}(\theta) d\theta \\
 &= \int_{BC1}^{TC} w_{cps,n}(\theta) d\theta + \int_{TC}^{BC2} w_{cps,n}(\theta) d\theta
 \end{aligned}
 \tag{9}$$

Fig.6 shows the power stroke portion of $w_{cps,n}(\theta)$ curve in Eq.9 before numerical integration is applied, along with the curves of $p_g(\theta)$ and $s_{p,n}(\theta)$ at $L = 3.40$. Since $w_{cps,n}(\theta)$ is the product of these 2 curves, the higher magnitudes and/or larger coincidence of both curves will yield higher and wider curve area, resulting in larger $W_{cps,n}$ integral value within the same power stroke. Obviously, the higher the $W_{cps,n}$ integral value, the more the work done, the larger the energy conversion benefit, thus the higher the engine efficiency.

The similar discussion can be made to engine torque, which is another important engine output parameter. Instantaneous torque $t_g(\theta)$ generated by gas pressure $p_g(\theta)$ is:

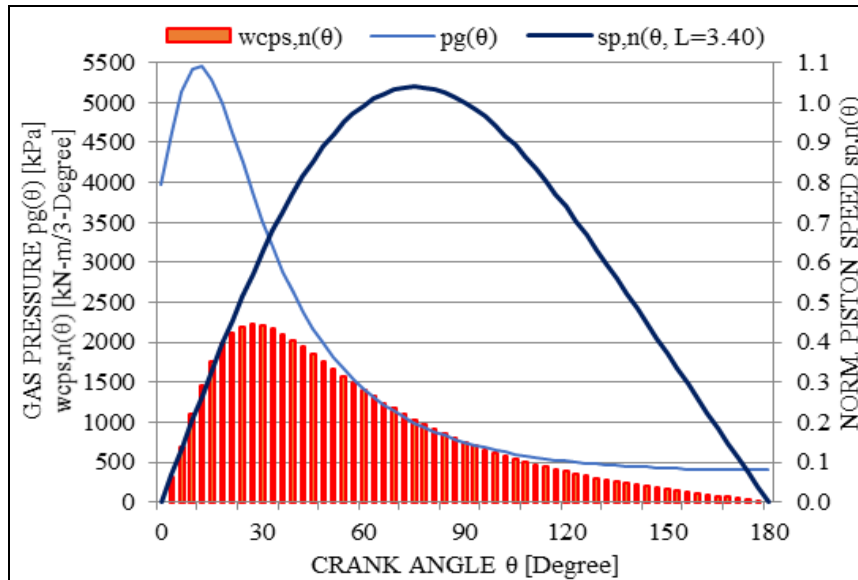


Fig 6: Curves of $p_g(\theta)$, $s_{p,n}(\theta)$, both at $L = 3.40$, and $w_{cps,n}(\theta)$

$$t_g(\theta) = [\text{Force_on_Piston}] \times [\text{Tangential_Torque_Arm_Length}]$$

$$= p_g(\theta) \cdot A_p \cdot r \cdot \sin(\theta) \cdot [1 + \cos(\theta) \cdot \frac{r}{l}] \tag{10}$$

where A_p is piston area, r is crank radius. Considering Eq. 2, Eq.10 can be converted to:

$$t_g(\theta) = p_g(\theta) \cdot A_p \cdot s_p(\theta) \tag{11}$$

Let $A_p = 1$, $r = 1$, then the normalized gas torque $t_{g,n}(\theta)$ becomes:

$$t_{g,n}(\theta) = p_g(\theta) \cdot \sin(\theta) \cdot [1 + \frac{\cos(\theta)}{L}] \tag{12}$$

$$t_{g,n}(\theta) = p_g(\theta) \cdot s_{p,n}(\theta) \tag{13}$$

Similar engine torque relationship is also given by Fayette ^[4] in its Eq. (8-50) which is relisted below

$$T_p(\theta) = p(\theta) \cdot A_p \cdot \frac{dS(\theta)}{d\theta} \tag{14}$$

where $T_p(\theta)$ is the torque caused by pressure $p(\theta)$, $S(\theta)$ is the piston travel distance. Thus $dS(\theta)/d\theta$ becomes the piston speed.

From Eq.13 and Eq.14, we may figure out that under the same cylinder pressure model $p_g(\theta)$, $t_{g,n}(\theta)$ is proportional to the magnitude of $s_{p,n}(\theta)$, where $s_{p,n}(\theta)$ works as a multiplier to $p_g(\theta)$, and could also be referred to as an equivalent of the coefficient of torque arm length. It is obvious that the higher the $s_{p,n}(\theta)$, the larger the $t_{g,n}(\theta)$, the higher the overall output torque.

5. Engine Mechanics Affecting Engine Efficiency

From Eq.9 and the above analysis, we know that engine mechanics affects engine efficiency in a large extent. The relationship between them is often handled in a macroscopic way that the variables are averaged upon multiple engine strokes and revolutions into “mean” values. Instead of in a microscopic way, Eq. 14 and Eq. 9 unveil the torque $t_{g,n}(\theta)$ and/or work $w_{ps,n}(\theta)$ depends on the interaction and coordination between cylinder gas pressure $p_g(\theta)$ and piston speed $s_{p,n}(\theta)$ within compression and power strokes in one engine cycle.

Using the microscopic way to investigate the relationship between engine mechanics and engine thermodynamics, we can identify the characteristics of reciprocal engines, which are helpful knowledge for engine efficiency improvement design.

As a multiplier in Eq.9 and Eq.13, piston speed behavior $s_{p,n}(\theta)$ plays an important role for engine work and torque output. The relationship revealed in these numerical models can be used to identify how to control piston speed behavior $s_{p,n}(\theta)$ to increase work output within working strokes for improving engine efficiency in a favorable direction.

Firstly, relative connecting rod length L affects normalized piston speed $s_{p,n}(\theta)$. According to Eq. 3, L is the primary parameter to $s_{p,n}(\theta)$. As can be seen from the curve family of $s_{p,n}(\theta)$ over L shown in Fig. 1, smaller L or shorter relative connecting rod results in larger peak value $S_{pmax,n}$ in $s_{p,n}(\theta)$. The values of $S_{pmax,n}$ over R in the range of (0, 1) are shown in Fig.3.

Secondly, relative connecting rod length L affects the maximum piston speed occurring angle or crank angle position θ_{spmax} . According to Eq.5, L is also a primary parameter to θ_{spmax} . As the curve of θ_{spmax} over L shown in Fig.2, the shorter the L , the smaller or earlier the θ_{spmax} . For regular engine with $L = 3.40$, the θ_{spmax} is at 75.19° .

Thirdly, smaller or earlier θ_{spmax} makes the curves of $s_{p,n}(\theta)$ lean toward the direction of TC ($\theta = 0^\circ$) along the axis of crank angle θ . Usually $p_g(\theta)$ generates peak cylinder pressure after TC and within 30° from TC. Thus the earlier θ_{spmax} , the higher coincidence degree $p_g(\theta)$ and $s_{p,n}(\theta)$ curves have, the larger product of $p_g(\theta)$ and $s_{p,n}(\theta)$, thus the larger integration result of $W_{ps,n}$ and overall $W_{cps,n}$. It should be noted that in the compression stroke, $W_{cs,n}$ is negative work. It is desirable to reduce the coincidence level between gas pressure $p_g(\theta)$ and piston speed $s_{p,n}(\theta)$ curves to reduce compression work consumed for achieving overall higher efficiency.

For regular engine with $L = 3.40$, $R = 0.294$, as shown in Fig.4, if R tends to be greater than 0.294, the θ_{spmax} tends to be smaller or earlier, resulting in larger coincidence of both $p_g(\theta)$ and $s_{p,n}(\theta)$ curves over the regular engine, thus the higher $W_{cps,n}$. Fig.4 has shown us the favorable direction for engine efficiency improvement.

From the above discussions about the effects of engine mechanics, we can conclude that engine work output within the compression stroke and the power stroke, $W_{cps,n}$ is influenced by engine mechanical configuration, in terms of relative connecting rod length L . Shorter L will yield more $W_{cps,n}$, thus higher engine efficiency.

6. Matching Gain and Cases Study

Utilizing the simulation models established in the previous sections, several cases in the engine development history are studied to identify the unique features of those engines, and to gain new insights into the leverage points and possibilities for engine efficiency improvement.

For comparison, a matching gain G_{bweps} is defined as the engine net brake output $W_{bweps,n}(L)$ for various engine configurations L or R over the baseline engine net brake output $W_{bweps,n}(L_0)$ within the enclosed compression stroke and power stroke.

The engine work output given by Eq.8 and Eq.9 is the indicated work. While the brake work can be calculated by subtracting friction work from to the indicated work:

$$W_{bweps,n} = W_{cps,n} - W_{friction} \tag{15}$$

Further, Eq.15 can be applied before the integration:

$$w_{bweps,n}(\theta) = w_{cps,n}(\theta) - w_{friction}(\theta) \tag{16}$$

For easy calculation and comparison, a mean friction work $w_{friction}$ is taken as fixed but different values during compression stroke and power stroke for all engines being compared:

$$w_{friction,comp} = 150 \text{ (kN-m/3-Degree)} \tag{17}$$

$$w_{friction,power} = 3 * w_{friction,comp} = 450 \text{ (kN-m/3-Degree)} \tag{18}$$

These values are derived from existing engine test data [3, 4], and they make good tradeoffs between various engine friction factors. Fig.7 shows the brake work curves for three types of engines for comparison.

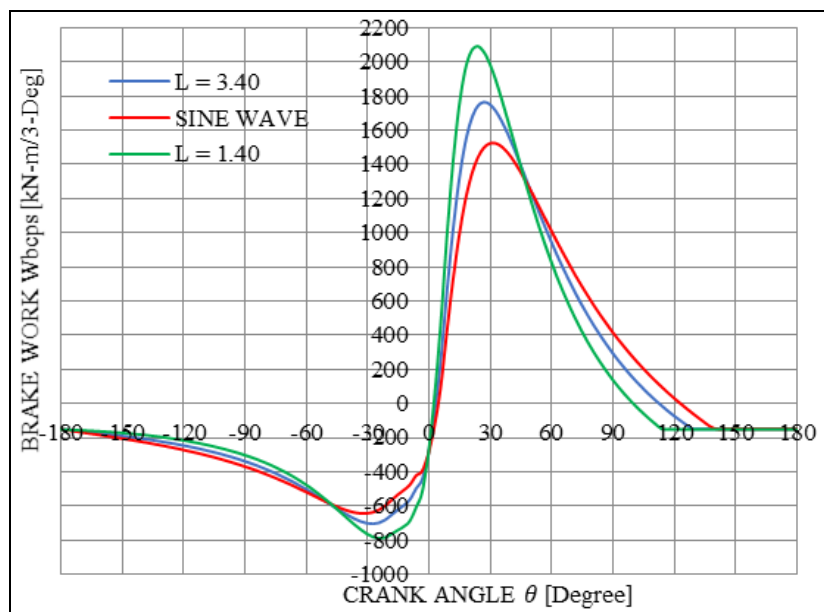


Fig 7: Engine Specific Brake Work $w_{bweps,n}$ Curves

The conventional crank-based production engine is taken as the baseline. In automotive industry, the average value of L is about 3.40, or $R = 0.294$. Taking $L_0 = 3.40$, and $R_0 = 0.294$, the matching gain G_{bwcp} could be calculated as the engine brake work output $W_{bcps,n}(L)$ for the engine with various L over the baseline engine brake work output $W_{bcps,n}(L_0=3.40)$ within the compression stroke and power stroke.

$$G_{bwcp} = \left[\frac{W_{bcps,n}(L)}{W_{bcps,n}(L_0=3.40)} - 1 \right] \times 100\% \tag{19}$$

$$G_{bwcp} = \left[\frac{W_{bcps,n}(R)}{W_{bcps,n}(R_0=0.294)} - 1 \right] \times 100\% \tag{20}$$

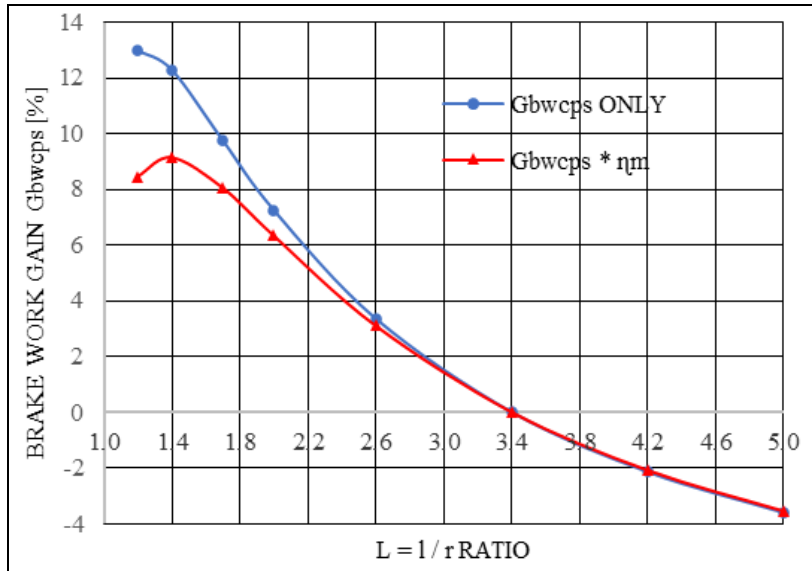


Fig 8: Matching Gain G_{bwcp} [%] over $L = 3.40$ Baseline

Fig.8 presents the calculated matching gain G_{bwcp} based on Eq.19 for various relative connecting rod length L . Positive gain can be achieved wherever L is smaller than 3.40. For example, $L = 1.40$ yields a gain of 12.28%. Once L is extended to greater than 3.40, matching gain G_{bwcp} becomes negative. For instance, when L is extended to 5.00, we would suffer a negative gain of -3.61% over the baseline engine.

Fig.9 shows the calculation results of Eq.20, where $R_0 = 0.294$ defines the baseline with zero matching gain. For a typical engine with $R = 0.294$, G_{bwcp} is over 10% higher than the engine with $R = 0$, the sine engine. As shown in Fig.4, these 2 engines have θ_{spmax} of 75° and 90° respectively. Yet 75° means some 15° leading 90° . Such a higher gain could be explained as 15° leading angle of θ_{spmax} reflects a better coincidence of $p_g(\theta)$ and $s_{p,n}(\theta)$ profiles in the typical conventional engine, and results in more work output and higher energy efficiency. This is one of the reasons that conventional crank-based engine structure has lasted and survived over 140 years and still being widely adapted nowadays.

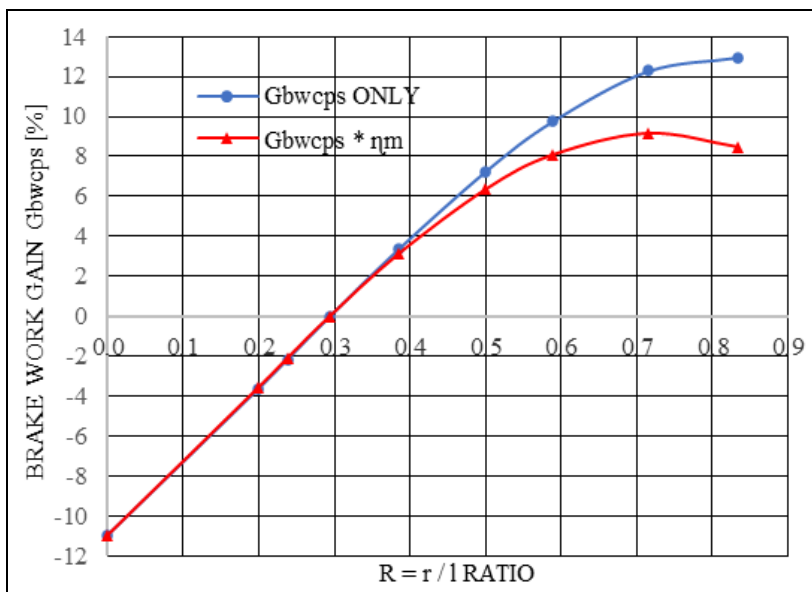


Fig 9: Matching Gain G_{bwcp} [%] over $R = 0.294$ Baseline

The model for engine kinematic efficiency given by Metange [8] can be adopted over the brake work gain calculation results in Fig. 8 and Fig. 9, in case we want to take into account of various L (inline crank mechanism) affecting on kinematic efficiency η_m , which drops from 100% to 50% as L changes from infinite to 1.00. The reason for such kinematic efficiency change could be explained with the friction caused by the increasing piston side-thrust force as L reducing. As a result, the upper curves in both Fig. 8 and Fig. 9 are pushed down to the lower ones, with a peak occurring at $L=1.40$ or $R=0.7143$.

Sine wave engine refers to any engine that runs mechanically in Sine law, with pure sinusoidal mechanical characteristics such as Bourke Engine [9] and many design variations based on Scotch yoke [10]. A negative matching gain of -10.98% for sine wave engine over the baseline engine with $R = 0.294$ can be seen in Fig.9 at the point of $R = 0$ where the engine mechanism is reduced to follow the Sine law. Such negative matching gain is due to the mismatching caused by 15° lag in Θ_{spmax} , which is believed the key limiting factor for this type of engines. Table 1 summarizes the parameters and result of 3 cases studied.

From the results listed in Table 1, it can be seen that engine efficiency is strongly related to engine mechanics. The mystery of these positive matching gains comes from the natural and primitive match between engine mechanics and engine thermodynamics.

Table 1: Case Study Parameters and Result

Parameters	Case 1	Case 2	Case 3
Engine Mechanism	Short Rod	Baseline	Sine Wave
Conn. Rod Ratio $L = l / r$	1.40	3.40	Infinity
Ratio $R = 1 / L = r / l$	0.714	0.294	0.000
Θ_{spmax} ($^\circ CA$)	63.96	75.19	90.00
Max. Piston Speed $S_{pmax,n}$	1.180	1.040	1.000
Overall Matching Gain (%)	12.28	0.00	-10.98

Offset engine, where the center line of crankshaft offsets to that of the cylinder bore, attracted much attention in history due to its various potentials. Using the design parameters of a typical offset engine reported by Shin *et al* [11], taking $L_0 = 2.96$ as the baseline in the report, a calculated matching gain of -2.734% is obtained based on Eq.19. This -2.723% gain is consistent with the test results reported [11] that no significant change could be observed because such a gain was hard to be measured and detected by the real measuring instruments.

Table 2: Offset Engine Parameters and Result

Parameters	Baseline	Offset
Crank Radius r (mm)	51.00	51.00
Crankshaft Stroke (mm)	102.00	102.83
Conn. Rod Length l (mm)	151.00	151.00
Conn. Rod Ratio $L = l / r$	2.961	2.961
Crankshaft Offset E (mm)	0.00	18.00
Piston BC1 ($^\circ CA$)	-180	-170
Piston TC ($^\circ CA$)	0	5
Piston BC2 ($^\circ CA$)	180	190
Compression Duration ($^\circ CA$)	180	175
Compression Θ_{spmax} ($^\circ CA$)	-73.53	-67.50
Compression Max. Speed $S_{pmax,n}$	1.051	1.107
Expansion Duration ($^\circ CA$)	180	185
Expansion Θ_{spmax} ($^\circ CA$)	73.53	80.37
Expansion Max. Speed $S_{pmax,n}$	1.051	1.023
Overall Matching Gain (%)	0.00	-2.723

Table 2 summarizes parameters and the above result of offset engine. Note the expansion duration has been extended from 180° to 185° , which makes the $s_{p,n}(\theta)$ curve wider but lower in $S_{pmax,n}$ and later in Θ_{spmax} , as shown in Fig.10, resulting in work gain of -1.34% in power stroke alone. On the other hand, compression duration has been squashed from 180° to 175° , which makes the $s_{p,n}(\theta)$ curve narrower but higher in $S_{pmax,n}$ and closer to TC in Θ_{spmax} , resulting in even lower overall $W_{bcps,n}$. That is why the overall matching gain has become negative according to Eq.9.

The reason of negative matching gain lies in the mismatching between engine mechanics and engine thermodynamics. It is clear that the mismatching between engine mechanics and thermodynamics decreases engine efficiency.

7. Optimization of Matching Between Mechanics and Thermodynamics

Based on the results from the above case studies, further analysis is made to identify the key factors affecting engine efficiency and to explore possible approaches and solutions for engine efficiency improvement.

A crucial feature is revealed in Eq.9 that $W_{cps,n}$ is the integration of the product of $p_g(\theta)$ and $s_{p,n}(\theta)$. Mathematically, the coincidence of their curves contributes to the integral value $W_{cps,n}$. Higher $p_g(\theta)$ and/or $s_{p,n}(\theta)$ magnitude will result in larger $W_{cps,n}$. For the same $p_g(\theta)$ and $s_{p,n}(\theta)$ magnitudes, the better the coincidence, the larger the $W_{cps,n}$ integration result. Physically,

for the same cylinder pressure, if being applied to the larger crank arm length, larger torque could be generated. Furthermore, as has been shown in Eq.10 through Eq.13, for the same cylinder pressure, if being applied alternatively to the piston moving faster or with higher $s_{p,n}(\theta)$, larger torque could also be generated.

From the above discussion, we can see that improving the coincidence between $p_g(\theta)$ and $s_{p,n}(\theta)$ functions will increase work gain. In other words, the better match between engine thermodynamics and engine mechanics will result in higher engine efficiency, given to the fact that $p_g(\theta)$ presents the characteristics of engine thermodynamics, while $s_{p,n}(\theta)$ presents the characteristics of engine mechanics. Refer to the curves in Fig.6, a perfect match requires that the peak of $p_g(\theta)$ curve leans rightward toward the peak of $s_{p,n}(\theta)$ curve, meanwhile, the peak of $s_{p,n}(\theta)$ curve leans leftward toward the peak of $p_g(\theta)$, as to obtain the maximum possible coincidence along the θ -Axis, as well as the maximum possible area under the curve of $w_{ps,n}(\theta)$.

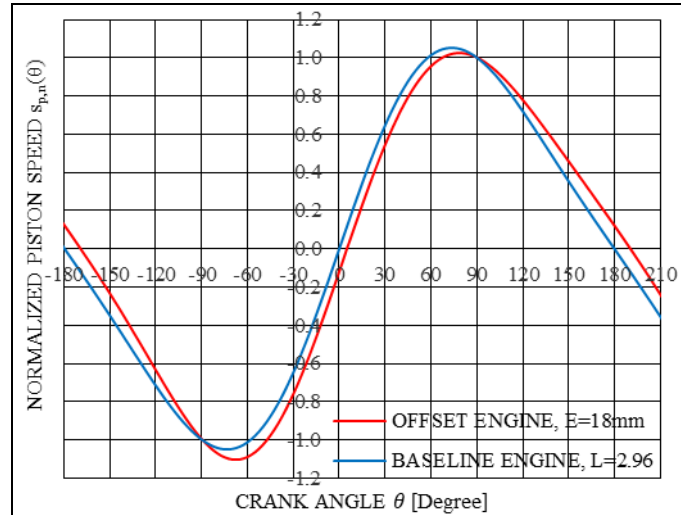


Fig 10: Normalized Piston Speed $s_{p,n}(\theta)$, Offset Engine and its Baseline

Beside power stroke, the discussions on the coincidence between engine mechanics and engine thermodynamics could be applied to compression stroke, so as to implement minimum possible work consumption for the compression. In this case, the matching variables should be handled in opposite way to those in the power stroke, i.e. reducing the magnitude of $s_{p,n}(\theta)$, and/or minimizing the coincidence of $p_g(\theta)$ and $s_{p,n}(\theta)$ curves during compression stroke.

Taking the curves in Fig.6 as a representative for typical engines in the market, there seems still a large room available for increasing the coincident degree between the two curves of gas pressure $p_g(\theta)$ and piston speed $s_{p,n}(\theta)$ in terms of the peaks, occurring time/angle and the curve shapes. The peak values of gas pressure $p_g(\theta)$ and piston speed $s_{p,n}(\theta)$ occur at crank angles of 12° and 75° respectively. The peak gas pressure is a decisive factor for engine material selection and dimensional design. However, only a partial and limited amount of the peak pressure generated during combustion is utilized for and contributes to engine work output.

From the current engine's coincident characteristic presented in Fig.6, we can see the potential for improvements in three directions:

1. Controlling the thermodynamic process to lean the peak of $p_g(\theta)$ curve rightward toward the peak of $s_{p,n}(\theta)$ curve;
2. Modifying engine mechanics to increase the peak value $S_{pmax,n}$ of $s_{p,n}(\theta)$ curve; and
3. Modifying engine mechanics to lean the peak of $s_{p,n}(\theta)$ curve leftward toward the peak of $p_g(\theta)$ curve.

For leaning the peak of $p_g(\theta)$ curve rightward toward the peak of $s_{p,n}(\theta)$ curve, let us consider an imaginary forced matching case. That is, keeping engine mechanics in $s_{p,n}(\theta)$ fixed and unchanged, controlling engine thermodynamics by forcing $p_g(\theta)$ moving rightward toward $s_{p,n}(\theta)$ along the θ -Axis with a phase delay angle Φ . Physically, this is equivalent to shift and relocate engine combustion process or shift and relocate phase of the starting point of power stroke, for trying to apply energy at the right timing and/or crank phase, targeting to achieve better matching. In fact, engines driven by compressed air or by bicyclists are used to do so.

Assuming combustion starting at crank angle Φ in the range of $(0^\circ, 90^\circ)$, we could estimate normalized work done by Eq.9 under various Φ , yielding

$$W_{ps,n}(\Phi) = \int_0^{BC} P_g(\theta) \cdot s_{p,n}(\theta) d\theta = \int_0^{BC} w_{ps,n}(\theta) d\theta \tag{21}$$

Next, let L be fixed at 3.40 and take $W_{ps,n}(\Phi)$ under $\Phi = 0$ as baseline, we obtain $W_{ps,n}(\Phi = 0)$. Then, calculate matching gain $G_{fpd}(\Phi)$ for the engine under forced phase delayed $p_g(\theta)$ profile over the baseline within the power stroke.

$$G_{fpd}(\Phi) = \left[\frac{W_{ps,n}(\Phi)}{W_{ps,n}(\Phi=0)} - 1 \right] \times 100\% \tag{22}$$

Fig.11 shows the calculation results of Eq.22, where $\Phi = 0$ defines the baseline zero matching gain. The value of $G_{\text{fpd}}(\Phi)$ means the matching gain over the conventional case of $\Phi = 0$. From Fig.11 positive matching gain can be observed wherever Φ is greater than 0° , with a maximum value of 59.11% around $\Phi = 45^\circ$ under $L = 3.40$.

The reasons for the optimal matching occurring around $\Phi = 45^\circ$ could be explained straightly. Once the cylinder pressure is applied onto the engine mechanism, the resulted peak torque would occur at 30° of crank angle after TC, as shown in Fig.6. Delaying this torque peak for $\Phi = 45^\circ$, it will be relocated to 75° of crank angle after TC. From Fig.2, Fig.4 or Fig.6 we know that the θ_{spmax} occurs at 75° after TC. Thus, the optimal matching is achieved when both the torque peak and the piston speed peak, or the peak of torque arm length, meet and coincide at 75° after TC.

As $p_g(\theta)$ curve is generated by the combustion process, and confined by the cylinder, piston and cylinder head / combustion chamber, we will face more challenges for manipulating $p_g(\theta)$ curve since more sophisticated modifications must be made to satisfy the constrains required by combustion process, which are not the focus points for this paper.

While $s_{p,n}(\theta)$ curve is determined by engine mechanics, that the engine designers could be able to modify it in various ways as to make $s_{p,n}(\theta)$ curve peaking higher and leaning more leftward toward $p_g(\theta)$ curve, as to obtain the maximum possible coincidence between them. For instance, H-Level engine proposed by Lazar ^[12] could move TC to 30° CA and $S_{\text{pmax},n}$ to 2.00 when the ratio of level length is 2.00. Reversely-pulled contra-rotary dual-crank engine developed by Islas ^[13] could shift TC up to 70° CA and $S_{\text{pmax},n}$ to 2.014 when $L = 3.128$ and crank offset $E = 2.00 \cdot r$.

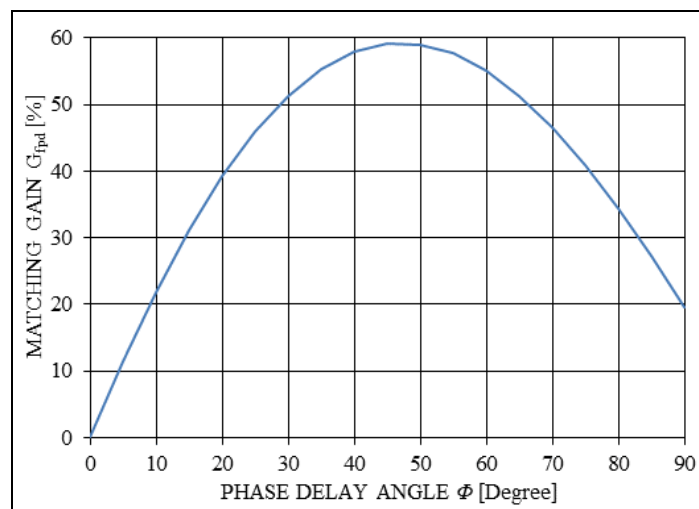


Fig 11: Forced Phase Delayed Matching Gain G_{fpd} [%], over $\Phi = 0$ and $L = l / r = 3.40$ Baseline

8. Conclusion

Engine mechanics determines thermodynamics process, while engine thermodynamics must be confined and determined by engine mechanics. The relationship between these two mating partners is established mathematically in this paper. The mismatching between them for typical engines available in the current market is identified, which also shed light for the possible directions for engine efficiency improvements. The concept of matching gain is defined and the method is developed for estimating the coincident level of engine mechanics and thermodynamics, which have been used for studying the engine development cases in the history, and the results interpretations bring new insights for engine efficiency improvements. The matching concept and the methodologies recommended could be used as a tool or a guideline for the designers to develop more efficient engines.

The optimizing process for matching would be the interactivity between engine mechanics and engine thermodynamics. Efforts have been made to design an engine with modified mechanics to achieve higher coincident level or matching degree between engine mechanics and thermodynamics. A novel engine model is being built for approaching optimal matching among all 4 individual strokes in a 4-stroke engine by Non-Circular gear train, and the description of this engine model and some analytical results will be presented in a separate paper ^[14]. A feasibility study is to be conducted for assessing the trade-off between the matching gain could be possibly achieved and the costs for the increased system complexity. Experimental validations are recommended.

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10. Appendix

For easy reference, the Acronyms and Abbreviations used in this paper are listed below:

BC	piston at Bottom Center point
BC1	intake stroke ending / compression stroke starting
BC2	power stroke ending / exhaust stroke starting
c	constant
G_{wps}	matching gain from power stroke work
G_{bwcp}	matching gain from compression and power stroke brake work
ICE	Internal Combustion Engine
L	ratio of con-rod length to the radius of crank
l	connecting-rod length
P	cylinder pressure
$p_g(\theta)$	instantaneous cylinder gas pressure, baseline engine
R	ratio of crank radius to connecting rod length
r	crank radius
$s_p(\theta)$	piston speed
S_{pmax}	maximum piston speed
$s_{p,n}(\theta)$	normalized piston speed, baseline engine
TC	piston at Top Center point
$t_g(\theta)$	instantaneous torque, baseline engine
V	cylinder volume
$X(\theta)$	piston position from piston pin to TC position
$x_p(\theta)$	position from piston pin to crank center
$W_{cs,n}$	work done over compression stroke
$W_{cps,n}$	work done over compression stroke and power stroke
$W_{friction}$	friction work
W_{ps}	work done over one power stroke
$W_{ps,n}$	normalized work done within one power stroke
γ	gas specific heat ratio
θ	crank angle
θ_{spmax}	crank angle of the maximum piston speed