International Journal of Advanced Research and Development
ISSN: 2455-4030
Impact Factor: RJIF 5.24
www.advancedjournal.com
Volume 2; Issue 6; November 2017; Page No. 581-592

R . C Matrices: A journey from 3-D to n-D

[1] Shukla Jaysan H, *[2] Dr. Pradeep J Jha
[1] Research Scholar, Rai University, Ahmedabad, Gujarat, India
[2] Professor of Mathematics, Rai University, Ahmedabad, Gujarat, India

Abstract
The rationale peripheral to the contents of this note is absolutely a new one that targets towards an area, not previously attempted one, in the vast field of matrix algebra. A new array of numerical figures, may be complex also, lies in the fact that in the matricinal construction wherein each row vector is always perpendicular to its corresponding column vector resulting in to an array/ matrix of three pair-wise perpendicular vectors. This system, in this note, has been treated in algebraic fairness with depicting a 3-D geometrical structure/s and their related characteristics. The extension to this finite system ends with enhancing glimpses to higher dimensions.

Keywords: perpendicular vectors, cone

1. Introduction
Sometimes it happens so that an object escapes away, not observing the centripetal force of attraction while whirling around some centre and its escaping tendency at that point of time is always tangential to the radius vector of the system. This particular system has summoned upon our attention and has envisaged our concentration in the direction. In addition to this there are, as observed in many celestial events, heard and read about, inspire us to think in the direction that we keep on unfurling some facts sequentially in this note. In this note we are to discuss about square matrices whose rows and column vectors observe orthogonality. This property, initially, does not shade fine colours but as we search for more mathematical details and algebraic structures it remains more sympathetic to what we pursue. As detail advances, we went on following and its pursuant was ceaseless which intuitively diverged us to the next paper on the same line as the title rightly indicates.

2. Definition of JJ (N) Set and "RC Property"
Now in this connection we define a square matrix M_n in n dimension with real entries (In remote cases we may extend the same to complex entries likely to be a part of wide open Hilbert space) we define, as narrated earlier, a square matrix $(n \times n)$ with each of its row vector denoted by $R_i;\forall i = 1,2,3,\ldots\ldots,n$ being perpendicular to each corresponding column vector $C_i;\forall i = 1,2,3,\ldots\ldots,n$ .The new chapter in this field that we are going to open begins with the most astounding properties for each matrix that we deal.

For each $i \in N$, we have the inner product of R_i and C_i denoted as $\langle R_i, C_i \rangle = 0$.

The general structure, depicted below, is in the form of an $(n \times n)$ array.

The general form of matrix possessing RC property is, what we have planned to align with to our further extension, as follows,

$$P = \begin{pmatrix} c_0 & a_1 & a_2 & & a_{n-1} \\ b_1 & c_1 & b_2 & \cdots & b_{n-1} \\ d_1 & d_2 & c_2 & & d_{n-1} \\ & \vdots & & \ddots & \vdots \\ m_1 & m_2 & m_3 & \cdots & c_{n-1} \end{pmatrix}_{n\times n}$$

The set of all such matrices is denoted as JJ (n). In this case, A $\in$ JJ (n)

3. Formation of Set JJ (3) with Real Entries
Now let us consider an element A of set JJ (3), with real entries

$$P = \begin{pmatrix} c_0 & a_1 & a_2 \\ b_1 & c_1 & b_2 \\ d_1 & d_2 & c_2 \end{pmatrix}$$

As a result of RC property, we have,

$$R_1 \cdot C_1 = 0 \Longrightarrow c_0{}^2 + a_1 b_1 + a_2 d_1 = 0$$
$$R_2 \cdot C_2 = 0 \Longrightarrow a_1 b_1 + c_1{}^2 + b_2 d_2 = 0$$
$$R_3 \cdot C_3 = 0 \Longrightarrow a_2 d_1 + b_2 d_2 + c_2{}^2 = 0$$

Let us consider cross terms by variables defined as below.

$$A = a_1 b_1, B = a_2 d_1, and\ C = b_2 d_2$$

which in turn allows us to rewrite,

$$R_1 \cdot C_1 = 0 \Longrightarrow c_0{}^2 + A + B = 0$$
$$R_2 \cdot C_2 = 0 \Longrightarrow A + c_1{}^2 + C = 0$$
$$R_3 \cdot C_3 = 0 \Longrightarrow B + C + c_2{}^2 = 0$$

Solution of this system of linear equations in terms of $c_0{}^2, c_1{}^2,$ and $c_2{}^2$, considering $A, B,$ and C as variables is,

$$A = a_1 b_1 = \frac{1}{2}[-c_0{}^2 - c_1{}^2 + c_2{}^2] \qquad (1)$$

$$B = a_2 d_1 = \frac{1}{2}[-c_0{}^2 + c_1{}^2 - c_2{}^2] \qquad (2)$$

$$C = b_2 d_2 = \frac{1}{2}[+c_0{}^2 - c_1{}^2 - c_2{}^2] \qquad (3)$$

So, every real element of set JJ (3) can be constructed by assuming its leading diagonal values c_0, c_1 and c_2. Also, solution shows that there exist infinite solutions for every choice of leading diagonal entries.

4. Construction of Set JJ (3) With Complex Entries

Generalization of Real formation can be obtained by taking complex element of JJ (3).
Let us consider, a complex element A of set JJ (3) by,

$$P = \begin{pmatrix} (a_1 + ia_2) & (b_1 + ib_2) & (c_1 + ic_2) \\ (x_1 + ix_2) & (y_1 + iy_2) & (z_1 + iz_2) \\ (p_1 + ip_2) & (q_1 + iq_2) & (r_1 + ir_2) \end{pmatrix}$$

By RC condition,

$$R_1 \cdot C_1 = 0 \Longrightarrow (a_1 + ia_2)^2 + (x_1 + ix_2)(b_1 + ib_2) + (c_1 + ic_2)(p_1 + ip_2) = 0$$
$$R_2 \cdot C_2 = 0 \Longrightarrow (x_1 + ix_2)(b_1 + ib_2) + (y_1 + iy_2)^2 + (z_1 + iz_2)(q_1 + iq_2) = 0$$
$$R_3 \cdot C_3 = 0 \Longrightarrow (c_1 + ic_2)(p_1 + ip_2) + (z_1 + iz_2)(q_1 + iq_2) + (r_1 + ir_2)^2 = 0$$

Let us consider, cross terms by variables as,

$$A = (x_1 + ix_2)(b_1 + ib_2), B = (c_1 + ic_2)(p_1 + ip_2), \text{and } C = (z_1 + iz_2)(q_1 + iq_2).$$
$$R_1 \cdot C_1 = 0 \Longrightarrow (a_1 + ia_2)^2 + A + B = 0$$
$$R_2 \cdot C_2 = 0 \Longrightarrow A + (y_1 + iy)^2 + C = 0$$
$$R_3 \cdot C_3 = 0 \Longrightarrow B + C + (r_1 + ir_2)^2 = 0$$

Solutions of this system of linear equations in terms of $(a_1 + ia_2)^2, (y_1 + iy)^2$ and $(r_1 + ir_2)^2$ is,

$$A = (x_1 + ix_2)(b_1 + ib_2) = \frac{1}{2}[-(a_1 + ia_2)^2 - (y_1 + iy_2)^2 + (r_1 + ir_2)^2] \quad (4)$$

$$B = (c_1 + ic_2)(p_1 + ip_2) = \frac{1}{2}[-(a_1 + ia_2)^2 + (y_1 + iy_2)^2 - (r_1 + ir_2)^2] \quad (5)$$

$$C = (z_1 + iz_2)(q_1 + iq_2) = \frac{1}{2}[+(a_1 + ia_2)^2 - (y_1 + iy_2)^2 - (r_1 + ir_2)^2] \quad (6)$$

So, every complex element of set JJ(3) can be constructed by assuming it's leading diagonal entries $(a_1 + ia_2), (y_1 + iy_2),$ and $(r_1 + ir_2)$. Also, solution shows that there exist infinite solutions for every choice of leading diagonal entries.

5. Case Studies of SET JJ (3)

Case-1: Leading diagonal elements are Integers

Let us take $c_0 = 1, c_1 = 2$ and $c_2 = 3$

Then by equations (1), (2), and (3), we can find A, B, and C as follows.

$$A = a_1 b_1 = \frac{1}{2}[-(1) - (4) + (9)] = 2$$

$$B = a_2 d_1 = \frac{1}{2}[-(1) + (4) - (9)] = (-3)$$

$$C = b_2 d_2 = \frac{1}{2}[+(1) - (4) - (9)] = (-6)$$

From infinite possibilities of A, B, and C let us consider following combinations

$A = a_1 b_1 = 2 \quad : a_1 = 1 \;\; \& \; b_1 = 2$
$B = a_2 d_1 = (-3) : a_2 = (-1) \;\; \& \; d_1 = 3$
$C = b_2 d_2 = (-6) : b_2 = (-1) \;\; \& \; d_2 = 6$

As a result we have $P = \begin{pmatrix} c_0 & a_1 & a_2 \\ b_1 & c_1 & b_2 \\ d_1 & d_2 & c_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 2 & -1 \\ 3 & 6 & 3 \end{pmatrix}$

Case-2: Leading Diagonal Elements are Proper Fractions

Let us take $c_0 = \frac{1}{2}, c_1 = \frac{1}{4}$, and $c_2 = \frac{1}{8}$.

Then by equations (1), (2), and (3) we can find A, B, and C as follow

$$A = a_1 b_1 = \frac{1}{2}\left[-\left(\frac{1}{4}\right) - \left(\frac{1}{16}\right) + \left(\frac{1}{64}\right)\right] = -\left(\frac{19}{128}\right)$$

$$B = a_2 d_1 = \frac{1}{2}\left[-\left(\frac{1}{4}\right) + \left(\frac{1}{16}\right) - \left(\frac{1}{64}\right)\right] = -\left(\frac{13}{128}\right)$$

$$C = b_2 d_2 = \frac{1}{2}\left[+\left(\frac{1}{4}\right) - \left(\frac{1}{16}\right) - \left(\frac{1}{64}\right)\right] = \frac{11}{128}$$

From infinite possibilities of $A, B,$ and C let us consider following combinations.

$$A = a_1 b_1 = -\left(\frac{19}{128}\right) : a_1 = (-19)\ \&\ b_1 = \frac{1}{128}$$

$$B = a_2 d_1 = -\left(\frac{13}{128}\right) : a_2 = \frac{1}{128}\ \&\ d_1 = (-13)$$

$$C = b_2 d_2 = \frac{11}{128} \quad : b_2 = \frac{1}{128}\ \&\ d_2 = 11$$

This helps write the matrix as shown. $P = \begin{pmatrix} c_0 & a_1 & a_2 \\ b_1 & c_1 & b_2 \\ d_1 & d_2 & c_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -19 & \frac{1}{128} \\ \frac{1}{128} & \frac{1}{4} & \frac{1}{128} \\ -13 & 11 & \frac{1}{8} \end{pmatrix}$

Case 3: Leading diagonal elements are Complex numbers.

Let us take $c_0 = \mathrm{i}, c_1 = -\mathrm{i},$ and $c_2 = 4\mathrm{i}$.

Then by equations (1),(2), and (3) we can find $A, B,$ and C as

$$A = a_1 b_1 = \frac{1}{2}[-(-1) - (-1) + (-16)\,] = (-7)$$

$$B = a_2 d_1 = \frac{1}{2}[-(-1) + (-1) - (-16)\,] = 8$$

$$C = b_2 d_2 = \frac{1}{2}[+(-1) - (-1) - (-16)\,] = 8$$

From infinite many possibilities of $A, B,$ and C let us consider following combinations.

$$A = a_1 b_1 = (-7) \quad : a_1 = (-1) \quad \&\ b_1 = 7$$
$$B = a_2 d_1 = 8 \quad : a_2 = 8 \quad \&\ d_1 = 1$$
$$C = b_2 d_2 = 8 \quad : b_2 = 2 \quad \&\ d_2 = 4$$

This helps write the matrix as shown. $P = \begin{pmatrix} c_0 & a_1 & a_2 \\ b_1 & c_1 & b_2 \\ d_1 & d_2 & c_2 \end{pmatrix} = \begin{pmatrix} \mathrm{i} & -1 & 1 \\ 7 & -\mathrm{i} & 4 \\ 8 & 2 & 4\mathrm{i} \end{pmatrix}$

6. Properties of Set JJ (3)

P(01): Let P be an element of set JJ (3) then its transpose denoted as P^T is also an element of set JJ (3).

Let us consider an element P of set JJ(3) as follows.

$$P = \begin{pmatrix} c_0 & a_1 & a_2 \\ b_1 & c_1 & b_2 \\ d_1 & d_2 & c_2 \end{pmatrix} \Rightarrow P^T = \begin{pmatrix} c_0 & b_1 & d_1 \\ a_1 & c_1 & d_2 \\ a_2 & b_2 & c_2 \end{pmatrix}$$

This matrix P^T also satisfies RC conditions which are same as that for A.

$$R_1 \cdot C_1 = c_0^{\,2} + a_1 b_1 + a_2 d_1 = 0$$

$$R_2 \cdot C_2 = a_1 b_1 + c_1^{\,2} + b_2 d_2 = 0$$

$$R_3 \cdot C_3 = a_2 d_1 + b_2 d_2 + c_2^{\,2} = 0$$

$\therefore P^T \in$ JJ(3)

P(02): JJ(3) is closed under scalar multiplication.

Let P be an element of JJ (3) and α is a scalar, then αP is also an element of JJ (3).
Let us consider an element P of set JJ(3) as follows.

$$P = \begin{pmatrix} c_0 & a_1 & a_2 \\ b_1 & c_1 & b_2 \\ d_1 & d_2 & c_2 \end{pmatrix} \Rightarrow \alpha P = \begin{pmatrix} \alpha c_0 & \alpha a_1 & \alpha a_2 \\ \alpha b_1 & c\alpha_1 & \alpha b_2 \\ \alpha d_1 & \alpha d_2 & \alpha c_2 \end{pmatrix}$$

This matrix αP also satisfies RC conditions which are same as that for P.

$$R_1 \cdot C_1 = \alpha^2 ({c_0}^2 + a_1 b_1 + a_2 d_1) = 0$$

$$R_2 \cdot C_2 = \alpha^2 (a_1 b_1 + {c_1}^2 + b_2 d_2) = 0$$

$$R_3 \cdot C_3 = \alpha^2 (a_2 d_1 + b_2 d_2 + {c_2}^2) = 0$$

$\therefore \alpha P \in$ JJ(3)

P(03): Let P be an element of set JJ (3) then its complex conjugate , denoted as P^{θ} is also an element of set JJ (3).
Let us consider a complex element P of set JJ(3) as defined below.

$$P = \begin{pmatrix} (a_1 + ia_2) & (b_1 + ib_2) & (c_1 + ic_2) \\ (x_1 + ix_2) & (y_1 + iy_2) & (z_1 + iz_2) \\ (p_1 + ip_2) & (q_1 + iq_2) & (r_1 + ir_2) \end{pmatrix}$$

This matrix will satisfy RC condition,

$$R_1 \cdot C_1 = 0 \Rightarrow (a_1 + ia_2)^2 + (x_1 + ix_2)(b_1 + ib_2) + (c_1 + ic_2)(p_1 + ip_2) = 0$$
$$R_2 \cdot C_2 = 0 \Rightarrow (x_1 + ix_2)(b_1 + ib_2) + (y_1 + iy_2)^2 + (z_1 + iz_2)(q_1 + iq_2) = 0$$
$$R_3 \cdot C_3 = 0 \Rightarrow (c_1 + ic_2)(p_1 + ip_2) + (z_1 + iz_2)(q_1 + iq_2) + (r_1 + ir_2)^2 = 0$$
$$R_1 \cdot C_1 = 0 \Rightarrow [(a_1)^2 - (a_2)^2 + (x_1 b_1 - b_2 x_2) + (p_1 c_1 - p_2 c_2)] + i\,[2a_1 a_2 + (x_1 b_2 + b_1 x_2) + (p_1 c_2 + p_2 c_1)] = 0 + i0$$
$$\Rightarrow [(a_1)^2 - (a_2)^2 + (x_1 b_1 - b_2 x_2) + (p_1 c_1 - p_2 c_2)] = 0 \text{ and } [2a_1 a_2 + (x_1 b_2 + b_1 x_2) + (p_1 c_2 + p_2 c_1)] = 0$$

$$R_2 \cdot C_2 = 0 \Rightarrow [(y_1)^2 - (y_2)^2 + (x_1 b_1 - b_2 x_2) + (q_1 z_1 - q_2 z_2)] + i\,[2y_1 y_2 + (x_1 b_2 + b_1 x_2) + (q_1 z_2 + q_2 z_1)] = 0 + i0$$
$$\Rightarrow [(y_1)^2 - (y_2)^2 + (x_1 b_1 - b_2 x_2) + (q_1 z_1 - q_2 z_2)] = 0 \text{ and } [2y_1 y_2 + (x_1 b_2 + b_1 x_2) + (q_1 z_2 + q_2 z_1)] = 0$$

$$R_3 \cdot C_3 = 0 \Rightarrow [(r_1)^2 - (r_2)^2 + ((p_1 c_1 - p_2 c_2) + (q_1 z_1 - q_2 z_2)] + i\,[2y_1 y_2 + (p_1 c_2 + p_1 c_2) + (p_1 c_2 + p_2 c_1)] = 0 + i0$$
$$\Rightarrow [(r_1)^2 - (r_2)^2 + ((p_1 c_1 - p_2 c_2) + (q_1 z_1 - q_2 z_2)] = 0 \text{ and } [2y_1 y_2 + (p_1 c_2 + p_1 c_2) + (p_1 c_2 + p_2 c_1)] = 0$$

Now,

$$\Rightarrow P^T = \begin{pmatrix} (a_1 + ia_2) & (x_1 + ix_2) & (p_1 + ip_2) \\ (b_1 + ib_2) & (y_1 + iy_2) & (q_1 + iq) \\ (c_1 + ic_2) & (z_1 + iz) & (r_1 + ir_2) \end{pmatrix}$$

$$\Rightarrow P = \overline{(P^T)} = \begin{pmatrix} \overline{(a_1 + ia_2)} & \overline{(x_1 + ix_2)} & \overline{(p_1 + ip_2)} \\ \overline{(b_1 + ib_2)} & \overline{(y_1 + iy_2)} & \overline{(q_1 + iq_2)} \\ \overline{(c_1 + ic_2)} & \overline{(z_1 + iz_2)} & \overline{(r_1 + ir_2)} \end{pmatrix}$$

$$\Rightarrow P^{\theta} = \begin{pmatrix} (a_1 - ia_2) & (x_1 - ix_2) & (p_1 - ip_2) \\ (b_1 - ib_2) & (y_1 - iy_2) & (q_1 - iq_2) \\ (c_1 - ic_2) & (z_1 - iz_2) & (r_1 - ir_2) \end{pmatrix}$$

For this matrix RC condition is,

$$R_1 \cdot C_1 = (a_1 - ia_2)^2 + (x_1 - ix_2)(b_1 - ib_2) + (c_1 - ic_2)(p_1 - ip_2)$$
$$R_2 \cdot C_2 = (x_1 - ix_2)(b_1 - ib_2) + (y_1 - iy)^2 + (z_1 - iz_2)(q_1 - iq_2)$$
$$R_3 \cdot C_3 = (c_1 - ic_2)(p_1 - ip_2) + (z_1 - iz_2)(q_1 - iq_2) + (r_1 - ir_2)^2$$

Using RC condition of matrix P,

$$R_1 \cdot C_1 = [(a_1)^2 - (a_2)^2 + (x_1 b_1 - b_2 x_2) + (p_1 c_1 - p_2 c_2)] \\ -i\ [2a_1 a_2 + (x_1 b_2 + b_1 x_2) + (p_1 c_2 + p_2 c_1)] = 0$$

$$R_2 \cdot C_2 = [(y_1)^2 - (y_2)^2 + (x_1 b_1 - b_2 x_2) + (q_1 z_1 - q_2 z_2)] \\ -i\ [2y_1 y_2 + (x_1 b_2 + b_1 x_2) + (q_1 z_2 + q_2 z_1)] = 0$$

$$R_3 \cdot C_3 = [(r_1)^2 - (r_2)^2 + ((p_1 c_1 - p_2 c_2) + (q_1 z_1 - q_2 z_2)] \\ -i\ [2y_1 y_2 + (p_1 c_2 + p_1 c_2) + (p_1 c_2 + p_2 c_1)] = 0$$

P(04): JJ (3) is closed under $"+"$, where operation $"+"$ is defined as usual matrix addition for any two elements P and Q of JJ(3), when $R_i \cdot C_i = 0$ for $i = 1, 2,$ and 3
i.e. R_i of P is perpendicular to C_i of P and Q for $i = 1, 2$ and 3.

Let us consider two elements A and B of JJ(3),then its addition as defined above will be,

$$P + Q = \begin{pmatrix} a_1 & b_1 & c_1 \\ x_1 & y_1 & z_1 \\ p_1 & q_1 & r_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 & c_2 \\ x_2 & y_2 & z_2 \\ p_2 & q_2 & r_2 \end{pmatrix} = \begin{pmatrix} a_1 + a_2 & b_1 + b_2 & c_1 + c_2 \\ x_1 + x_2 & y_1 + y_2 & z_1 + z_2 \\ p_1 + p_2 & q_1 + q_2 & r_1 + r_2 \end{pmatrix}$$

Then it will satisfy RC property as,

$$R_1 \cdot C_1 = [(a_1)^2 + (a_2)^2 + (x_1 b_1 + b_2 x_2) + (p_1 c_1 + p_2 c_2) \\ +2a_1 a_2 + (x_1 b_2 + b_1 x_2) + (p_1 c_2 + p_2 c_1)]$$

$$= 0 \quad (\because R_1 \text{ of P is perpendicular to } C_1 \text{ of P and } C_1 \text{ of Q})$$

$$R_2 \cdot C_2 = [(y_1)^2 + (y_2)^2 + (x_1 b_1 + b_2 x_2) + (q_1 z_1 + q_2 z_2) \\ +2y_1 y_2 + (x_1 b_2 + b_1 x_2) + (q_1 z_2 + q_2 z_1)]$$

$$= 0 \quad (\because R_2 \text{ of P is perpendicular to } C_2 \text{ of P and } C_2 \text{ of Q})$$

$$R_3 \cdot C_3 = [(r_1)^2 - (r_2)^2 + ((p_1 c_1 - p_2 c_2) + (q_1 z_1 - q_2 z_2) \\ +2y_1 y_2 + (p_1 c_2 + p_1 c_2) + (p_1 c_2 + p_2 c_1)]$$

$$= 0 \quad (\because R_3 \text{ of P is perpendicular to } C_3 \text{ of P and } C_3 \text{ of Q})$$

$\therefore P + Q \in$ JJ (3) This establishes closure property.

P (05): JJ (3) is associative under addition.
Let P, Q, and R be elements of JJ (3) then,
Using the RC properties described above one can established

$\therefore P + (Q + R) = (P + Q) + R$

P (06): Existence of an additive identity in JJ (3).
Null matrix O_3 is an additive identity element of JJ (3).
Followed by RC property, Null matrix is also a member of JJ (3) and it acts as an additive identity in JJ (3).

$\therefore P + O = O + P$

[This has been granted as a special case.]

P (07): Existence of an additive inverse in set JJ (3).
$i.e.$ For any element P of JJ (3) there exists an element $(-P)$ in JJ (3) such that

$P + (-P) = (-P) + P = O.$

P (08): JJ (3) is commutative under addition.
$i.e. P + Q = Q + P$ For any element P and Q of JJ (3).

$$P + Q = \begin{pmatrix} a_1 & b_1 & c_1 \\ x_1 & y_1 & z_1 \\ p_1 & q_1 & r_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 & c_2 \\ x_2 & y_2 & z_2 \\ p_2 & q_2 & r_2 \end{pmatrix} = \begin{pmatrix} a_1 + a_2 & b_1 + b_2 & c_1 + c_2 \\ x_1 + x_2 & y_1 + y_2 & z_1 + z_2 \\ p_1 + p_2 & q_1 + q_2 & r_1 + r_2 \end{pmatrix}$$

As all the entries of matrices P and Q are real and the commutative property holds true in the set R we have,

$$P + Q = \begin{pmatrix} a_2 + a_1 & b_2 + b_1 & c_2 + c_1 \\ x_2 + x_1 & y_2 + y_1 & z_2 + z_1 \\ p_2 + p_1 & q_2 + q_1 & r_2 + r_1 \end{pmatrix} = \begin{pmatrix} a_2 & b_2 & c_2 \\ x_2 & y_2 & z_2 \\ p_2 & q_2 & r_2 \end{pmatrix} + \begin{pmatrix} a_1 & b_1 & c_1 \\ x_1 & y_1 & z_1 \\ p_1 & q_1 & r_1 \end{pmatrix} = Q + P$$

$\therefore P + Q = Q + P$

P (09): JJ (3) is an Abelian Group.
Properties P (04), P(05), P(06), P(07), and P(08) assert that JJ (3) is an Abelian group.

P (10): Elements of JJ (3) are non-singular
As R_i is only perpendicular to C_i for each $i = 1,2,$ and 3 vectors represented by R_i are non-planner. So the underline matrix is always non-singular matrix. [Null matrix is one special case of JJ (3)]

P (11): Existence of multiplicative identity of JJ (3).
As the rows and columns of regular identity matrix do not obey RC property we claim that Regular identity matrix is not a member of JJ(3) and hence this implies multiplicative inverse of a member JJ(3) vulnerable inverse.

P (12): Existence of multiplicative inverse of JJ (3).
As multiplicative inverse of members of JJ(3) does not exists we can find a matrix that possesses characteristics which parallel to an inverse matrix this inverse matrix, if it exists, will be known as a maximal inverse corresponding to a particular member of JJ(3).

7. Graphical representation of an element of set JJ (3)
In this section we consider the geometrical meaning of our basic matrix satisfying "RC Property" [i.e. $R_i \cdot C_i = 0$ for $i = 1,2,3.$] As, $\overrightarrow{R_1} \perp \overrightarrow{C_1}$, we have in R^3 space, two vectors which are orthogonal to each other and in general $|R_i| \neq |C_i|$, we get two vectors $\overrightarrow{R_1}$ and $\overrightarrow{C_1}$ showing two perpendicular line segmentswhich in turn can be extended to the construction of a rectangle with origin as co-terminus point of $\overrightarrow{R_1}$ and $\overrightarrow{C_1}$. With this fact we can construct one of its diagonal as $\overrightarrow{R_1 + C_1}$

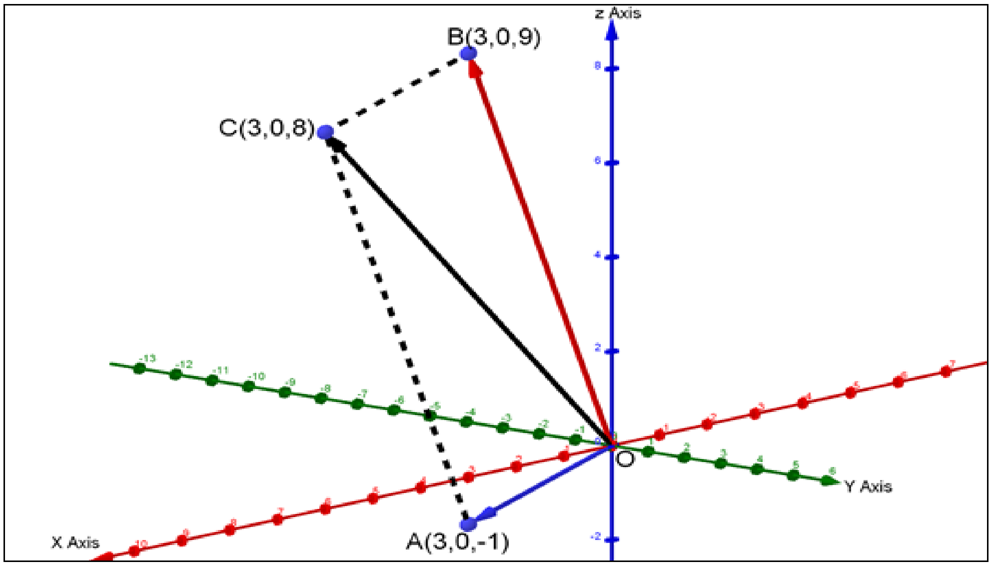

Fig 1

As shown in the above figure 1, $\overrightarrow{R_1} = \overrightarrow{OA}$ and $\overrightarrow{C_1} = \overrightarrow{OB}$ and hence $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$ as $\overrightarrow{OB} \parallel \overrightarrow{AC}$ now we are equipped with what we want to convey. We have two cases.

Case - I: If we consider $\overrightarrow{OC}$ as the axis and follow the vector $\overrightarrow{OA}$ to rotate along, we get a conical structure as shown in figure 2.

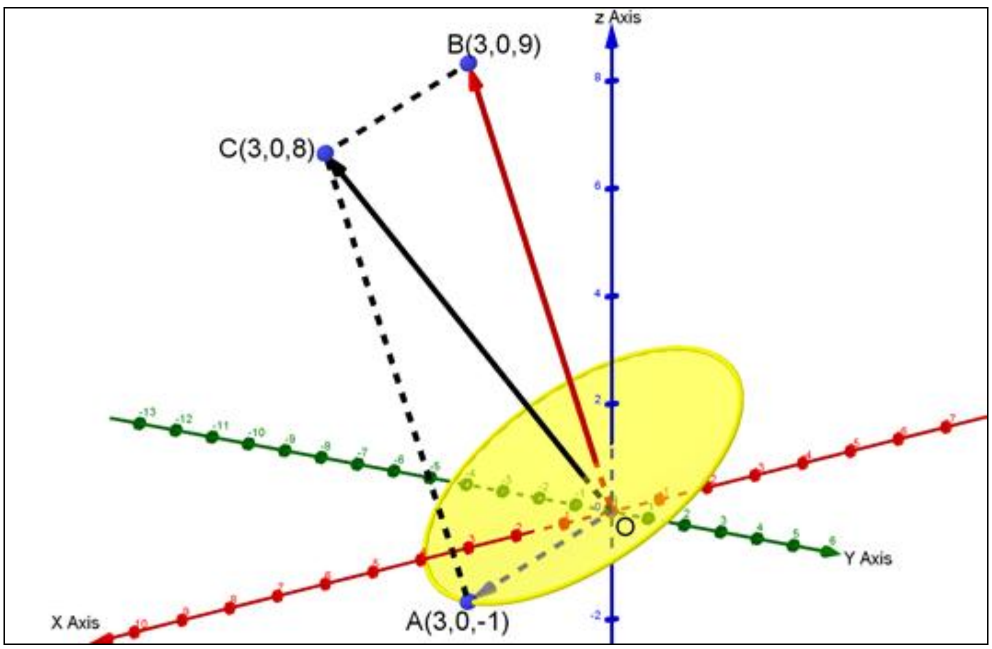

Fig 2

Case - II: In the same line, rotating $\overrightarrow{OB}$ along $\overrightarrow{OC}$, we get another conical structure. As it corresponds to $\overrightarrow{R_1}$ and $\overrightarrow{C_1}$, we call these two canonical structures as the first companion canonical structure CC_1.As shown in in figure-3

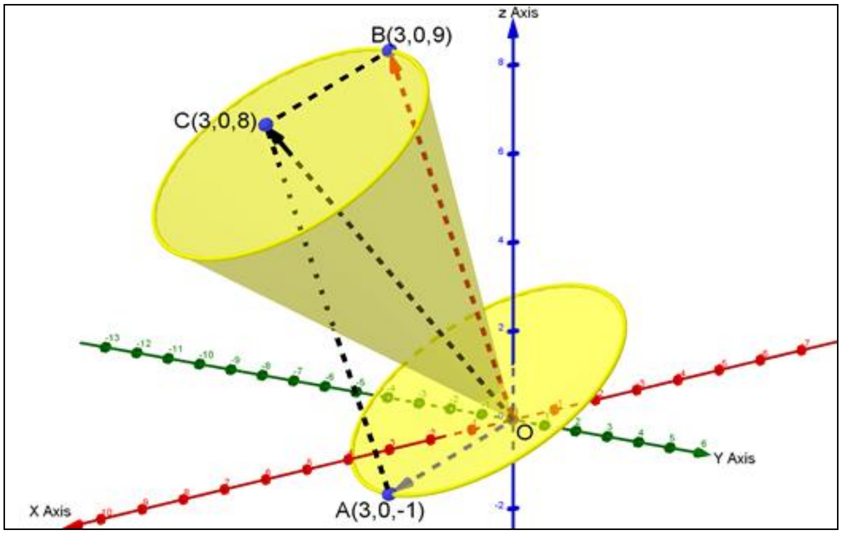

Fig 3

Continuing efforts in the same direction we have two more sets of canonical structures corresponding to $\overrightarrow{R_2}$ and $\overrightarrow{C_2}$, as the second companion canonical structure CC_2 along with $\overrightarrow{R_3}$ and $\overrightarrow{C_3}$,as the third companion canonical structure CC_3.As shown in figure-4,

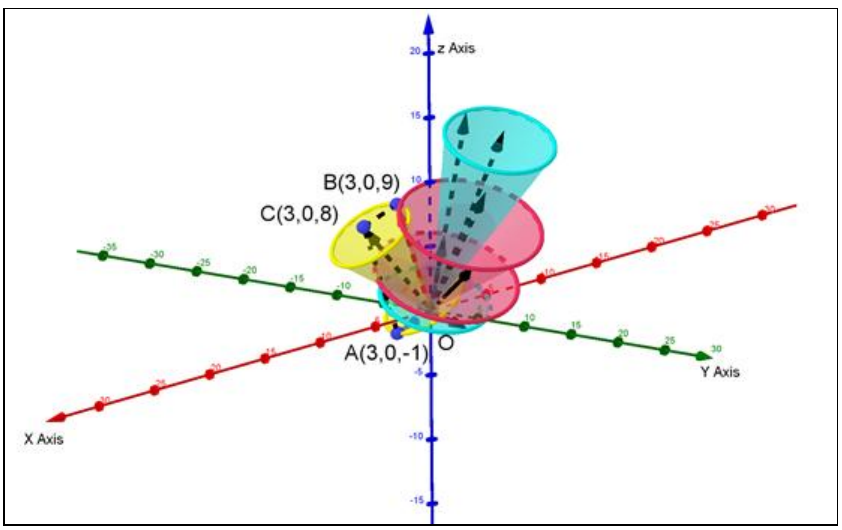

Fig 4

Thus, we have led to conclusion from the defining properties that the RC matrix with real entries permits us to perceive the underlying matrix as a set of 3 pairs of companion cones as $CC_1, CC_2$and CC_3.

We proceed on the same lines to a finer resolution. We start with a matrix possessing RC property and intrinsically consider all real entries for the purpose. It will be, later on, extended to on a complex field. The general form of the RC matrix on hand is,

$$P = \begin{pmatrix} a & b & c \\ x & y & z \\ p & q & r \end{pmatrix}$$

Following in built property of RC matrix we have three simultaneous equations in nine variables which are,

$$R_1 \cdot C_1 = 0 \Longrightarrow (a)^2 + bx + cp = 0 \quad (7)$$

$$R_2 \cdot C_2 = 0 \Longrightarrow bx + (y)^2 + zq = 0 \quad (8)$$

$$R_3 \cdot C_3 = 0 \Longrightarrow cp + zq + (r)^2 = 0 \tag{9}$$

Let us denote,
$A = bx, B = cp$ and $C = qz$ with A B and C not all zero at a time[i.e. $A^2 + B^2 + C^2 \neq 0$.
[In the case violating above condition we have a null matrix-a special case of RC matrix]
From equations (7), (8), and (9) in terms of above mentioned replacements, we have,

$$a = \pm\sqrt{-(A+B)} \tag{10}$$

$$y = \pm\sqrt{-(A+C)} \tag{11}$$

$$r = \pm\sqrt{-(B+C)} \tag{12}$$

Which to preserve real system rolls down to three conditions, which are as follows.

$(A+B) < 0, (A+C) < 0$ and $(B+C) < 0$ which is equivalent to $A < -B$ and $B < -C$

Again it's noteworthy that $A = bx$ for fixed real values throws an infinite choice for selection of values of $'b'$ and $'x'$. parallel arguments for B and C continue for choices of variables c, p, q and z.
We now focus on the previous argument $A = bx$ where A a real positive number allows infinitely many combinations for b and x where both b and x possessing same sign.
[In the case $A < 0$ and $(A+B) < 0$, we have the case with opposite signs for both b and x]
In any case, we have for a fixed A and given $b \neq 0$ $x = \frac{A}{b}$ and this situation when interpreted graphically gives rise to a rectangular hyperbola.s
Concluding on the parallel reasoning for each one of B and C also we have a rectangular hyperbola. This conveys the message that each one of RC matrix is presentation of one point on each rectangular hyperbola.
Thus a RC matrix designs a set of three corresponding rectangular hyperbolas.
The notable fact is that once the rectangular hyperbolas and it's equations are formed from a given 'RC' matrix we can, using the equations of these rectangular hyperbolas, construct infinite 'RC' matrices.

8. Converse to the Above Fact
In this section we derive the converse to the above fact without loss of generality. It is claimed that if we three different real constants; say c_1, c_2, and c_3 which correspond to three different rectangular hyperbolic system xy = a real constant; [in this case each one of c_1, c_2, and c_3] then for each pair of point like (x, y) on each one of the rectangular hyperbola, we have to had three pairs of points satisfying the equations and which in turn are solvable for infinite values. [Say for a fixed C_1, we have $b_i x_i = c_1$ for different values of natural i. $c_i p_i = c_2$, and $z_i q_i = c_3$]. These values in accordance with equations (10), (11), and (12) give the real entries of 'RC' matrix and such are infinite in number.
Without leaning into rigor of the subject on hand, we satisfy by exemplifying a numerical.

Given three rectangular hyperbolas are,

$XY = c_1$, $XY = c_2$, and $XY = c_3$.

For construction of member of JJ(3), let us consider these rectangular hyperbolas as,

$A = bx = c_1$
$B = cp = c_2$
$C = qz = c_3$
If we select points on rectangular hyperbolas

$A = bx = c_1$
$B = cp = c_2$
$C = zq = c_3$

Randomly as (1, c_1),(1, c_2)and (1, c_3) respectively. For simplicity let us assume,

$b = c_1$ & $x = 1$
$c = c_2$ & $p = 1$
$z = c_3$ & $q = 1$

We can construct a member of JJ(3) as,

$$P = \begin{pmatrix} a & b & c \\ x & y & z \\ p & q & r \end{pmatrix}$$

This satisfies RC property as,

$$R_1 \cdot C_1 = 0 \Longrightarrow (a)^2 + bx + cp = 0 \Longrightarrow a = \sqrt{-(bx + cp)} \Longrightarrow a = \sqrt{-(A + B)}$$

$$R_2 \cdot C_2 = 0 \Longrightarrow bx + (y)^2 + zq = 0 \Longrightarrow y = \sqrt{-(bx + zq)} \Longrightarrow y = \sqrt{-(A + C)}$$

$$R_3 \cdot C_3 = 0 \Longrightarrow cp + zq + (r)^2 = 0 \Longrightarrow r = \sqrt{-(cp + zq)} \Longrightarrow r = \sqrt{-(B + C)}$$

In particular if given three rectangular hyperbolas are,

$XY = 2, XY = -3$ and $XY = -6$.

For construction of member of JJ (3) let us consider these rectangular hyperbolas as,

$bx = 2$
$cp = -3$
$qz = -6$

If we select points on rectangular hyperbolas $bx = 2$, $cp = -3$ and $qz = -6$
Randomly as (1, 2), (-1, 3) and (-1, 6) respectively.
As shown in Figure.5

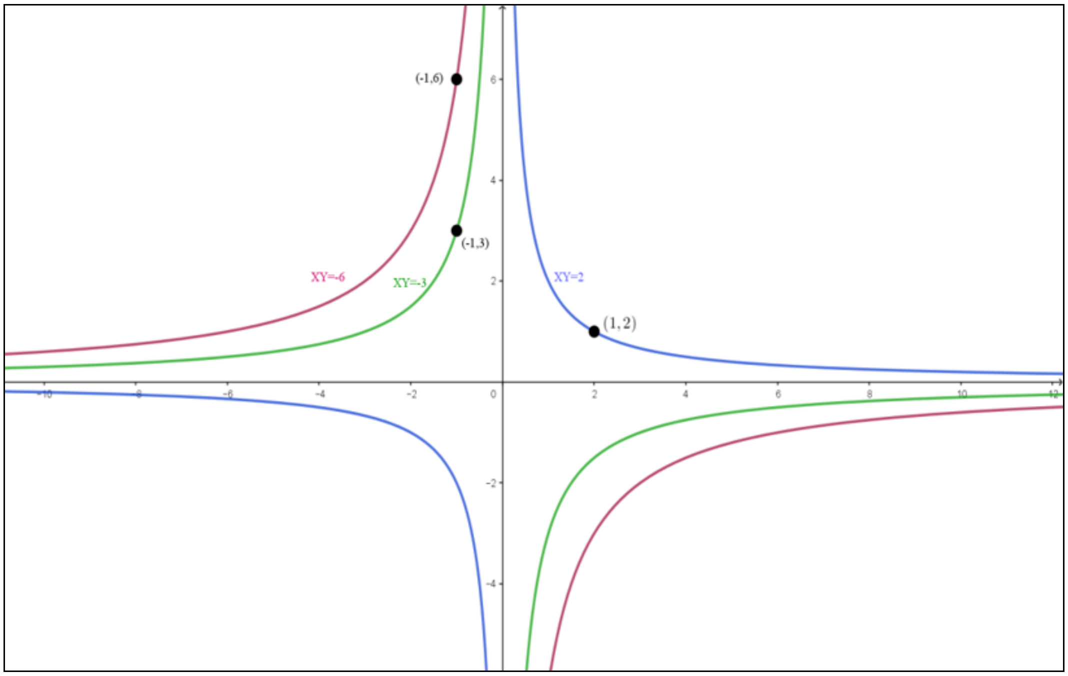

Fig 5

We can construct a member of JJ (3) as,

$$P = \begin{pmatrix} a & b & c \\ x & y & z \\ p & q & r \end{pmatrix} = \begin{pmatrix} a & 2 & 3 \\ 1 & y & 6 \\ -1 & -1 & r \end{pmatrix}$$

This satisfies RC property as,

$$R_1 \cdot C_1 = 0 \Longrightarrow (a)^2 + bx + cp = 0 \Longrightarrow a = \sqrt{-(bx + cp)} \Longrightarrow a = \sqrt{1} \Longrightarrow a = \pm 1$$

$$R_2 \cdot C_2 = 0 \Longrightarrow bx + (y)^2 + zq = 0 \Longrightarrow y = \sqrt{-(bx + zq)} \Longrightarrow y = \sqrt{4} \Longrightarrow a = \pm 2$$

$$R_3 \cdot C_3 = 0 \Longrightarrow cp + zq + (r)^2 = 0 \Longrightarrow r = \sqrt{-(cp + zq)} \Longrightarrow r = \sqrt{9} \Longrightarrow a = \pm 3$$

So we can construct, on using the data derived from three rectangular hyperbolas, $2^3 = 8$ different RC matrices. They are derived from the given root matrix P and shown as follows.

$$A_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 6 \\ -1 & -1 & 3 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & -2 & 6 \\ -1 & -1 & 3 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 6 \\ -1 & -1 & -3 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & -2 & 6 \\ -1 & -1 & -3 \end{pmatrix}$$

$$A_5 = \begin{pmatrix} -1 & 2 & 3 \\ 1 & 2 & 6 \\ -1 & -1 & 3 \end{pmatrix} A_6 = \begin{pmatrix} -1 & 2 & 3 \\ 1 & -2 & 6 \\ -1 & -1 & 3 \end{pmatrix} A_7 = \begin{pmatrix} -1 & 2 & 3 \\ 1 & 2 & 6 \\ -1 & -1 & -3 \end{pmatrix} A_8 = \begin{pmatrix} -1 & 2 & 3 \\ 1 & -2 & 6 \\ -1 & -1 & -3 \end{pmatrix}$$

[These all matrices observe RC property.]

Conclusion

What we found at the end of profound thinking and pertaining to physical significance of our views has finally tracked us on an unknown path, we thought initially, but ceaseless efforts have lead us to an extent of utmost satisfaction by involving us in rigorous version of mathematics. In addition to that we achieved, has opened avenues of higher dimensions.

Vision

We plan to extend and enhance this notion gradually to higher dimensions. We hope that the extension on this will break the barriers of human visibility and drag a mathematical minded existence to open gates to envisage higher dimension for which cognitive domain generally does not allow to pierce in.

References

1. Hill FS, Jr. The Pleasures of 'Perp Dot' Products." Ch. II.5 in Graphics Gems IV (Ed. P. S. Heckbert). San Diego: Academic Press. 1994, 138-148.
2. Davis PJ. The mathematics of matrices: A First Book of Matrix Theory and Linear Algebra, gim-blaisdell, 1965.