International Journal of Advanced Research and Development
ISSN: 2455-4030, Impact Factor: RJIF 5.24
www.newresearchjournal.com/advanced
Volume 1; Issue 6; June 2016; Page No. 36-45

Exploring the effect of manipulatives on students' performance

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Abstract

This Research is a consolidated approach towards the improved and result-oriented development of mathematical understanding in children from an early age with the major role contributed by the use of manipulatives. Manipulatives significantly help children with Picturisation and imagination, resulting in enhanced comprehension from a very early age of mathematics and numbers. If children are exposed to clear, smart, and thought-out manipulatives at an early school grade, such as Kindergarten, their entire academic performance can be improved multifold, increasing the average IQ level of a primary and secondary level student. However, not much in-depth research has been conducted on this subject. This study aims to prove the positive results achieved by using manipulatives for learning purposes when it comes to mathematics, and how with a little more focus and research, new techniques and details can be found that can further the early development of children at every age.

Keywords: manipulatives, children, mathematics, research, understanding mathematics, activity-based learning

Introduction

Chapter 1

"A boring lecture can extinguish the flames of everyone's fire" (Hook, 1994, P. 37), there is a strong belief that manipulatives have a treasured place in the understanding of mathematics. Active participation and involvement increase students' learning. Incorporating this concept in mathematics is arduous, for the mere fact that mathematics is so "abstract." One way of enhancing students' comprehension and interest in mathematics is the use of manipulatives. Teachers in the primary and secondary schools have accepted and recognized the significance of manipulatives (Hartshorn *et al*., 1996). And recent research on students' learning of mathematical conceptions and processes have corroborated the fact that more and more students have shown massive interest and appreciate mathematics when teachers use manipulatives to explain the concepts.

According to the National Council of Teachers of Mathematics (2001) manipulatives is a must have hands-on in every grade levels. According to Hartshorn *et al*. (1996) manipulatives are beneficial in assisting children to move from the abstract to the concrete level. Manipulatives are objects that can be moved or positioned; this strategy calls for acting out a problem by using models and the likes to explain mathematical concepts. There are many different ways of physically representing a problem; traditionally, physical representations have been left out of the mathematics curriculum of most schools, thus making math totally abstract to some people. However, when objects represent problems, they become concrete and more easily understood (Herr & Johnson, 2001).

When planning a manipulative approach, you often have to make a conscious effort to look for a way to do the problem using manipulatives you must have a little inventiveness in you; the necessary materials are often right around you. For example, dimes, and pennies make excellent manipulatives; not only are they of different sizes and different colors, but also the backside of each can be distinguished from the front side. If you are in the forest, you can always find twigs, leaves, and pebbles (to represent wolves, bears, and campers, for example). The trick is to think of using them and then to use them effectively. Sometimes the first manipulative doesn't work well, so it may be effective to create a new one. Suydam and Higgins (1977) explained that mathematics performance improves when manipulatives are used. Higgins (1977) examined the potency of different kinds of manipulatives with elementary and secondary school students and concluded that continuous use of manipulatives was more potent than limited use. Higgins also discovered that countless teachers are not making use of manipulatives. Hence, the study urges teachers to explore the use of manipulatives.

Significance of the study

Manipulatives helps in nurturing children's power of logic, creativity, abstract or critical thinking, enhances problem-solving abilities and improves even their communication skills. This study is important for several reasons. First, understanding the impact of manipulatives in teaching mathematics can help in expounding the underlying logic of understanding mathematical concepts and can help parents, students as well as schools evaluate strategies that makes mathematic interesting. To expand students' mathematical dexterities, teachers and leaders should routinely incorporate the use of virtual and concrete manipulatives into classroom teaching (Suydam & Higgins, 1977).

Significance of the study to teachers

As Tsoukas (2005) puts it, the more human beings know, the more able they will be to control their destiny." The majority of today's leaders and teachers have realized that to function successfully nowadays, it is necessary to become knowledge-based. Knowledge is a source of sustainable competitive advantage and must have ability for the creation of values for the next generation of society (Nonaka & Nishiguchi, 2001). The outcome of this study may provide reliable and useful information for teachers globally. Knowledge gained from the outcome of this study may strengthen the breadth of manipulatives (mathematics) as a branch of knowledge that can produce positive scholarly insight. The outcome may also suggest avenues by which teachers could receive more fulfillments from their efforts and more cooperation from their students.

Roles (Importance) of manipulatives

Manipulatives play a very significant role in enhancing understanding of mathematics. Manipulation of symbols and objects enables students who are terrified and often drop out of math class because of their inadequate application of number sense. (Hartshorn, Boren & Sue, 1996). Physically touching objects provide students with the right sense of interpretation, especially the geometric interpretation of symbols and objects. Thus, they enhance students' understanding as they visually make connections between various object. Additionally, manipulatives enhance collaborative study, and help improves discussion or dialogue in the algebra class by furnishing students' objects to think with and deliberate on (Hartshorn, Boren & Sue, 1996). Manipulative materials like pattern blocks and Cuisenaire rods have a great impact on children's learning as they enable children to investigate mathematical and scientific ideas (like shapes and number) by actual manipulation of real life objects (Resnick &Martin, 1996). As students' work, reflection, and manipulate the objects, the real learning takes place.

Factors affecting the use of manipulatives

Availability is perhaps the greatest factor affecting the use of manipulatives; obviously, teachers cannot utilize what is unavailable. However, teachers can improvise by simply collecting and making do with what is available such as buttons, and spools, shapes, colors, bean-sticks, etc., that are easy to make. Another factor affecting the use of manipulatives is that teachers sometimes express concern that sharing and cleanup of manipulative materials are laborious (Lappan *et al*., 1996).

Application of geometric objects such as tiles, cubes, geoboards, dice, and counters has exponentially increased students understanding of geometry. These objects accord students the opportunity of continual exploration by creating, sketching, and discussing different challenging situations. Sadly, there has not been extensive use of manipulatives in the secondary schools. Therefore, the study on their effectiveness at this level is very little (Howden, 1986). Howden, in an attempt to make a connection between abstract and concrete through the use of manipulative used tiles in the classroom to help transition students' from abstract to the concrete level in algebra. The tiles prototype addresses the fundamental concepts of polynomials, from definition to factoring and multiplying polynomials where he emphasized the idea of linking geometry to algebra allowing children to employ prior knowledge to the subject matter. As students' uses the tiles, they were encouraged to draw pictures and to make out rational images. This strategy was implemented during this research in my classroom (details on prediction result), and students were able to achieve ninety percent (90%) mastery.

Research assumptions (Hypotheses)

Hypothesis is a formal statement of the anticipated correlation(s) between two or more variables in a stated or named population, beliefs that may or may not be accurate (Macnee, 2004). In this study, the types of hypothesis used are simple and directional. The hypotheses used are as follows:

- **Assumption 1:** Students' interest in learning mathematics will increase.
- **Assumption 2:** Students will make a significant improvement in their geometric and algebraic thinking skills.

Definition of key terms

- **Manipulative:** tangible objects; things or devices that can be handled or hold, and moved around by students to initiate or buttress a mathematical idea. Semi-concrete level: a portrayal of a real circumstance, where images or portraits of the actual objects are used instead of the items themselves.
- **Semiabstract level:** a metaphorical depiction of concrete items.
- **Geo-board:** a device for drawing two-dimensional shapes. It provides excellent methods for children to draw shapes.
- **Static visual representation:** visual images mainly associated with pictures that can be shown on an overhead projector or sketched on a chalkboard that allows students to flip, slide, and rotate an object (manipulate) as desired.
- **Dynamic visual representation:** visual objects or images on the computer much like pictures in books, sketches on an overhead projector, and on a chalkboard.
- **Virtual manipulative:** an interactive, Internet-based visual portrayal or depiction of dynamic objects that offers opportunities for building mathematical skills.
- **Physical representation:** is a strategy for solving problems using physical or actual objects.
- **Systematic Lists:** a list created through some form of structure.
- **A System:** is any set of rules that allows people to do something (like organize information) in an orderly fashion.
- **Kinesthetic learners:** individuals who learn best by carrying out physical activities through moving and touching rather than listening or watching demonstrations.

Chapter 2

Literature review

The following are detailed accounts of what has been published on the topic "Using manipulatives" to enhance understanding of mathematics in the classroom. In this study, Manipulative is referred to as tangible objects; things or devices that can be handled or hold, and moved around by students to initiate a mathematical concept. According to Hartshorn, Robert-Boren, and Sue, (1996), vigorous involvement boosts students learning, and one practical route for bringing experience to bear on students' mathematical comprehension is hands-on approach such as manipulatives. This method has been corroborated by the National Council of Teachers of Mathematics (NCTM) which has expressly and openly encouraged the use of manipulatives in every grade levels. NCTM agrees that manipulatives are particularly useful in assisting struggling students' of mathematics transition from the abstract to the concrete level. That is, from theoretical to physical or symbolic form. The problem with Hartshorn, Robert-Boren, and Sue, (1996) findings was their claim that teachers barely use manipulatives, but the reasons for such lack was not given.

There is a great connection between manipulatives and a better understanding of mathematical concepts. In his book, "Elementary and middle school mathematics—teaching developmentally," Van De Walle (2001) offered a compelling idea for using manipulatives to enhance understanding of mathematics which collaborates with the work of others who have studied the importance of manipulatives in understanding mathematics. In the research Van De Walle (2001) exposes so many strategies for understanding math concepts; among them was electronic manipulatives for numeration, a mathematical software tool, much like a real life manipulative with which to investigate mathematical concepts (Van De Walle, 2001). With this software, students can drag objects and figures to anywhere on the screen, alter or adjust the colors, and arrange them in groups. Van De Walle (2001) also explained that electronic manipulatives apart from keeping students' interested and engaged could give students options and the power to expand or minimize the size of blocks visually by "set increments." In that same study, Van De Walle (2001) talks about using electronic manipulatives to measure areas, perimeters and to rotate as well as reflect an object and creating three-dimensional shapes.

Another numeration tool Van De Walle (2001) exposed was a "hundreds of boards" which allows students to color in squares with the click of a mouse and search for patterns. It gives users (students) the following advantages.

- Freedom of manipulation—students must be encouraged to use the replica in their way without undue guidance or feedback from the computer.
- Connection with symbolism
- Journal capability—this program offers a space at the bottom of the screen where students can write about their work. Teachers may be able to use this same space to present problems to students as well as solving the problems.
- Print and save capability—it is easy for students to print a picture of their work, keep their works, and or return to their jobs at a later time.
- The ability to expand or minimize the size of an object through set increments
- The power to "glue" objects together to form new ones
- The power to reflect or rotate objects across a reflection line or a point
- The ability to compute or calculate area or perimeter of an object
- Capacity to choose geometric figure with a variable number of sides
- The power to create three-dimensional shapes and possibly rotate them in space.

In addition to these features, Van De Walle (2001) explains that with electronic manipulatives for numeration, sketching of shapes on a grid is easier and functional for geometric exploration than "free-form drawing." A shape like a triangle can easily be altered with the use of rubber band on a geoboard to form another shape. The electronic geo-board program provides a larger grid on which to draw and is easy to use and can be printed and saved for later use. Electronic manipulatives enable students' perform mathematical operations that seem difficult or impossible with physical geo-board (Van De Walle, 2001).

Similarly, Resnick *et al*. (1998) validate Van De Walle's explanations on electronic manipulative by shading more lights on digital manipulative materials like beads, blocks, badges, and balls. According to Resnick *et al*. (1998), these manipulative materials play a significant role in students' learning, as they enable students' explore scientific and mathematical concepts (like numbers and shapes) by personally and physically manipulating objects. Using these manipulative materials, children develop better perspectives on mathematical concepts such as shape, size, and number (Resnick *et al*., 2004). According to Resnick *et al*. (2004), there are countless mathematical concepts which are hard (if not impossible) to explore with standard or conventional manipulative materials. As he puts it, traditional manipulatives mostly do not help students' grasp or master concepts that are associated with dynamics and systems, in view of the fact that they are taught through more formal methods that involve manipulation of abstract symbols, rather than physical objects that are accessible only to older students with better mathematical skill. Therefore, with this knowledge and insight, digital manipulative was developed or designed to make concepts salient for students (Resnick *et al*., 2004).

Among these salient concepts are, programmable bricks which give students the power to create and control. Programmable Bricks has output ports for controlling motors and lights, and input ports for acquiring information from sensors (light, touch, and temperature). P-Bricks enable students to write a logo program on a personal computer, download the program to the P-Brick and store it in the computer where students can continually modify and customize its behavior (Resnick *et al*., 2004). Programmable brick provides deep connections to the abstracts and is undoubtedly an excellent approach to learning. With this concept, students will begin grasping the fundamental concept of patterns, shapes, sizes, and most importantly it helps in capturing students' attention and keep them interested and on task. As students' engage in these activities, it gives them the chance to interact and communicate with one another, asking questions whenever they get confused (Resnick *et al*., 2004).

Beads

This small, colored piece of plastic allows students to create colorful patterns. In the same research Resnick *et al*. (2004) made use of what he called "programmable Beads" to engage and stimulate students by letting them create dynamic patterns using Beads. These programmable Beads have a "microprocessor and Light-Emitting Diode" (LED) that enables them to communicate with one another through "inductive coupling" that beads simultaneously in different ways to produce different dynamic patterns of light. Programmable Beads can excite or inspire students to begin thinking about probabilities (Resnick *et al*., 2004).

As Resnick *et al*. (2004) put it, students who grew up playing with programmable Beads develops deeper intuitions and better geometric understanding faster than students who were not exposed to programmable Beads. Thus, there is a notion that understanding mathematical concept centers around the proper and efficient use of manipulatives (Spikell, 2005). In his study, Spikell (2005) discussed "Virtual manipulatives" and goes a step further to discuss two kinds of virtual manipulatives----Static and dynamic visual depictions of concrete manipulative. In his view, a virtual manipulative is best explained as hands-on, web-based visual illustration of a dynamic object that provides an opportunity for creating mathematical skills which correlate with what most authors on manipulatives observed. Spikell (2005) agrees with others that manipulatives are very useful in schools for problems such as tangrams, geo-board, pattern blocks, fraction bars, and geometric solids. Using virtual manipulatives enables students to make as many pattern blocks as they desire to create designs in both vertical and horizontal style (Dorward & Heal, 1999). This interactive ability allows the children to take part in and regulate the physical operations of these objects combined with the opportunities to find and formulate mathematical principles and relationships differentiates them as virtual manipulatives (Dorward & Heal, 1999). The advancement in today's technology has improved and made this application easy and less complicated, students can be looking at the screen and use the mouse to point, click, and drag objects. No doubt, with the advancement of technology there may be other ways to move virtual manipulative in future; perhaps through features like "voice commands and infrared signals" (Spikell, 2005).

People gain a new perspective and insight into a problem when using objects or when acting out the problem (Johnson & Herr, 2001). The next few pages are devoted to problem-solving strategies, namely: manipulatives and using people to act out the problems. As Johnson and Herr (2001) explained, physical representation has traditionally been left out of the mathematics curriculum of most schools because mathematics many people perceive them to be abstract. Therefore, when problems are presented with objects, they become concrete and more easily understood (Herr, 2001). Johnson and Herr Findings collaborate with the work of others who have studied extensively with the use of manipulatives to enhance understanding of mathematics. Manipulatives play a significant role in algebra courses as well. They furnish ways that students who are not comfortable with algebra could manipulate symbols. Manipulatives also accord students a clearer and better interpretation of geometric symbols, enriches students' understanding by helping them make a powerful connection with other subjects other than mathematics (Sowell, 1989). Encouraging and supporting cooperative learning through manipulatives materials and group activities, providing students' objects to think with, talk about, and reflect on, fosters quick learning and retention skills (Picciotto, 2005).

In his article "What Role Can Manipulative Play in the Classroom" Alejandre (2005) inferred that individual students learn differently and this difference in learning is accentuated with the use of manipulative. Alejandre, like other reputable researchers on the subject of manipulatives, maintains that manipulatives are paramount in the development of mathematical concepts because as students touch and move objects, they intuitively formulate and develop other ways of looking at mathematics. Apart from meeting the needs of students who learn best in this way, teachers who use manipulatives can learn new ways of writing a topic (Alejandre, 2005). Also, Alejandre (2005) explains that a sound lesson on any mathematical topic should involve multiple instructional techniques. Integrating various instructional methods increases the likelihood that every student can develop mathematical understanding through at least one method. Offering students an activity with three components like (activities, technology and formalizing) will not only give students with different learning style multiple ways of looking at the problem but will allow them the extra time they may need for learning and mastery (Alejandre, 2005). In essence, time and experience in class enriche activities and students can learn from their experiences and connect the mathematics to those experiences. Such a foundation helps them to understand and appreciate mathematics (Alejandre, 2005). Williams (1998) said it better when he said:

"Tell me math and I forget; show me math and I may remember; involve me and I will understand mathematics. If I understand mathematics, I will be less likely to have math anxiety. And if I become a teacher of mathematics, I can thus begin a cycle that will produce less math anxious students for generations to come" (p. 101).

Manipulatives in conjunction with other techniques enrich and strengthens students' comprehension; however, depending solely on them as a means of impacting knowledge can also be detrimental and ineffective (Alajander, 2005). Simply using only manipulatives without formal discussion, abstraction, and mathematical connection could make students lose the opportunity of deeper conceptual learning (Alejandre, 2005). The research by Bellonio (2012) is an interesting study one that coincides with what Suzanne Alejandre articulated in her article. Bellonio (2012) invokes the old Chinese proverb, "I hear, and I forget, I see, and I remember, I do, and I understand." Bellonio maintains that active participation strengthens and boosts students' learning and applying the, I hear, and I forget, I do, and I understand the idea of mathematics by way of manipulatives will bring experience to bear on students' mathematical comprehension. Bellonio (2012) sees manipulatives as inexpensive, simple to use objects that enhance students' mathematical concepts.

It is a known fact that students learn better by active participation than when they are not actively engaged. According to Spikell (2005) most learners, be them, children or adults, grasps mathematical concepts and skills quicker when they are offered either in the "concrete/pictorial or symbols." Using manipulatives, symbols, and pictures to interpret or depict abstract ideas help learners understand the abstractions they represent (bellonio, 2012). Not too long ago while teaching 7^{th} and 8^{th} grade mathematics, I presented my students with puzzles consisted of squares that were divided into seven pieces. The students were given an opportunity to explore the puzzle, reorganize the pieces to formulate new designs and shapes. After playing with the puzzle pieces, a student explained that it felt as if he was back in elementary. This student was so happy that working with such material reminded him how his elementary school teacher used to explain concepts that made them easier to comprehend.

Research also shows that continuous use of manipulative is far better than periodic use. According to Bellonio (2012) many students (and some adults) have math anxiety, thus always discouraged by poor performance; poor math related experiences and their inability to comprehend mathematics. These students do not appreciate nor discern or perceive the need for its value or its applicability in the real world. This math anxiety usually leads students to settle with careers where there is no or limited application of mathematics Bellonio, 2012). Mathematics is one discipline that is virtually everywhere, therefore, as long as our society advances so also are the desire for mathematics skills (Bellonio, 2012). In his study, Howard Gardner specifies eight unique intelligences, two of those ways can be taught, and he referred to them as "linguistic intelligence and logical intelligence." People who are linguistically gifted have natural or intrinsic love for language. They are endowed with good vocabularies and are eloquent in expression, illustration, writing; reading, listening, editing and speaking (Bellonio, 2012). The logically-mathematically gifted people take pleasure in finding or creating concepts, looking for patterns and relationships, and doing activities in a sequential order. They like to organize the steps of a piece of work or task into a sensible order and need time to finish each piece. These people relish every opportunity they get to solve a problem, explore new things, working with games, puzzles and kits, collecting organizing information, tasks and delight in pastimes (Bellonio, 2012). According to Gardner (1983) though there is no one best technique of teaching and testing, however, employing manipulatives usually bring students to an equitable level when it comes to comprehension.

In the same vein, Gardner (1983) talked about the kinesthetic learner, one of the eight distinct bits of intelligence and infers that kinesthetic students/learners are those individuals who learn through touching, moving, and doing. Usually, they are not receptive of visual or auditory instructions but are willing and able to confront problems with high activity. They enjoy the role-playing stimulation, manipulative, physical exercise, competitive sports, games and action-packed activities (Gardner, 1983). These groups of learners are attracted to manipulatives because they can feel, and touch the objects, so, teaching with manipulatives boosts their retentive abilities and comprehension of mathematics (Gardner, 1983, p. 34). Also in the study, Gardner discovers that manipulative help relieves boredom in students, as it offers a change from the abstract learning allowing students to explore and use their imagination (Gardner, 1983). As Gardner puts it, it is profitable or advantageous for teachers to be conscious of the various types of learners and plan lessons in ways to reach and appeal to them. If a student is taught in a style that does not appeal or suitable with his or her learning style, it might cause the student great anxiety toward that particular subject, thus that student may not make the best or most effective use of the learning process (Gardner, 1983, p. 35).

Therefore, it is imperative that math classes be taught in various ways. Games like Math Bingo where teachers can ask varieties of assessment-type questions like areas, angles, circumference, and the calculation of money are ways teachers can reach students (Bellonio, 2012). Teachers can also integrate a Quizmo-type card game where students are provided with cards that hash questions on one side and answers on the other side, and they are to listen for the question and match it with a reply. Activity that calls for measurement of actual objects or items like fractions, patterns geometric shapes and symmetry are also useful in skill reinforcement (Bellonio, 2012). Also, science and art projects that utilize graphs or developing patterns using geometric figures could be appealing to some students and help them make a smooth transition from math to science or arts, even play with computers or similar technologies can be beneficial to many students (Bellonio, 2012).

Ideally, the fact that most mathematics teaching activities occur at the abstract level, every mathematics concept should be initiated at the communication level, because it is assumed that students have the propensity to overlook or ignore anything taught only in abstraction stage that makes students frustrated and loses interest mathematics because of their failure to attain mastery (Bellonio, 2012). In linking "abstract" to the "concrete," Bellonio gave examples of those objects or symbols that could represent or used as manipulatives. They includes calculators, money (paper or coin), paper clips, two-color counters, buttons, took picks, graph papers, playing cards, strings, number cubes (dice), rulers, capacity containers, drinking straws, spinners, thermometers, pattern blocks, dominoes, geo-board, pentomonoes, etc that could be effectively utilized in the classroom. These objects could be used to coach and explain concepts like estimation, percent, decimals, probability, fractions, whole numbers, counting, measurement, prime numbers, angles, geometry, place value and factoring (Bellonio, 2012).

In addition to the above listed, games are excellent means of allowing students to apply prior knowledge to the real world situations. Games are very inspirational to students and could be utilized to ameliorate particular skills, using games in the classroom and at home could maximize students' problem-solving skills as well as their ability to think logically and communicate mathematically (Bellonio, 2012). Teachers can use manipulative to clarify concepts and vocabularies associated with each lesson plan. Activities involving manipulative have been found to lead to a deeper understanding of mathematics. Boyle in his article entitled "Reading, Writing, & Learning in ESL" maintains that manipulative encourages students to ask questions which can lead to a further understanding of the mathematical process (Ma, 1999). Questioning reinforces academic instruction, regardless of whether the subject is math or language. Regrouping, subtracting, and borrowing all refer to one process; manipulative makes students slow down and prove their mental progression. Another advantage of using manipulative Stupiansky *et al*, pointed out is that manipulatives give the students a chance to think critically about the process instead of simply going through the motions (Waite-Stupiansky, Sandra, Stupiansky & Nicholas, 1998).

Another area of study for which manipulatives proved effective is the power of doing. According to (Reutzel & Cooter, 2000) manipulatives provides students with concrete experiences to develop a concept. At the same time, teachers can guide oral language development as students work with hands-on tools. The number one principle for effective vocabulary instruction is that vocabulary is learned best through direct, hands-on experience (Reutzel & Cooter, 2000). An example of how this can be achieved is to cover a rectangular region with square tiles and then use the three-dimensional figure to discuss the concept of the area with students. Dividing the process into a series of steps allows students to work independently. After the discussion, students should have a more concrete understanding of the area as well as the necessary (associated) vocabularies (Reutzel & Cooter, 2000).

Likewise, researchers found manipulatives to be highly effective in the lives of students in the area of molding their understanding, particularly in geometric shapes. Students can use manipulatives to find the circumference and diameter of different sized circles using strings. Reutzel and Cooter (2000) explains that during this activity, students can be provided with the definitions of circumference and diameter while at the same time given the opportunity to explore the concepts themselves. Providing the definitions to the students while simultaneously engaging them in a hands-on activity ensures that students a more concrete assimilation of the concept and its application than they would receive reading the definition in their mathematics textbook (Tepper, 1999).

Furthermore, Reutzel and Cooter (2000) explains the significance of manipulative to English language learners (ELLs), they mentioned that math concepts could be more meaningful for English Language Learners if it is embedded in lessons and linked to the real life situations that they (ELLs) have experienced. Math concepts such as addition, subtraction, multiplication, division, and fractions can be illustrated through the use of tangible objects such as cubes, buttons, beans, and pattern blocks. Application of these objects makes math concepts more meaningful and accessible English Language Learners and further the lesson, students could be encouraged to find examples of geometric shapes in their homes and classroom. Everyday objects such as cereal boxes can be used as examples of rectangles. Other demonstrations include the following: stop signs can illustrate octagonal shape, slices of pizza can be used to teach fractions, and egg cartons can illustrate halves, fourths, thirds, and sixths (Monroe & Panchyshyn, 1995-1996).

Chapter 3
Experimental methods

In exploring this topic, as with any research topic, there is much thought that needs to go into the planning process. The setting, population, methods, and data collection are all vital aspects of this process. With that thought in mind, the following chapter will address those issues.

Setting and population

This research project occurred in one of the Middle Schools in the Detroit metropolitan area. The school educates 800 students from Kindergarten through eighth grades. This school has a large population of culturally diverse students and staffs. There is also a wide range of socioeconomic backgrounds. At this Middle School, Mathematics is not an elective course, it is a must course for all students. Students attend mathematics classes' every day for forty-five minutes. The groups of students chosen to participate in this ten-week study are thirty (30) students from my sixth-grade classes and thirty students from my seventh-grade classes for a total of sixty (60) students, male and female. Students were chosen because of their limited knowledge of basic concepts in dealing with Geometry and algebra. In the 2004-2005 school year, a new curriculum was written for the middle school mathematics department, called "algebraic thinking" and "algebraic foundation" for sixth and seventh-grade students' with a strong emphasis on the use of manipulatives. At this School, mathematics has an important place in every subject areas.

Methods

During a two-month period, the students were exposed to as many different manipulative materials as possible. Geo-boards, Acting it out by (making a systematic list), and Visual (electronic) manipulatives were some of the techniques used on this project.

The first area of study was the use of geo-boards. Group discussions were held on what information the students already knew about geo-board. Students were shown how to find the area and perimeter of any polygon by using a geo-board and rubber bands. The geo-board is constructed with raised points to enable the students to design polygons. For example, to make a triangle, the rubber band is arranged around the number of points to make the shape. A demonstration on how to find the area of a triangle, which is the space within the triangle or the covering of the triangle measured in square units, was performed. Here students need to know the base and the height of the constructed triangle. To do this, we took one-half of the base and multiplied it by the height. Also, we used the Pythagorean Theorem (a^2 + b^2 = c^2) to find the length of the slanted sides. For the perimeter—the total distance around the object or the surrounding of an object, written (L + W x 2).

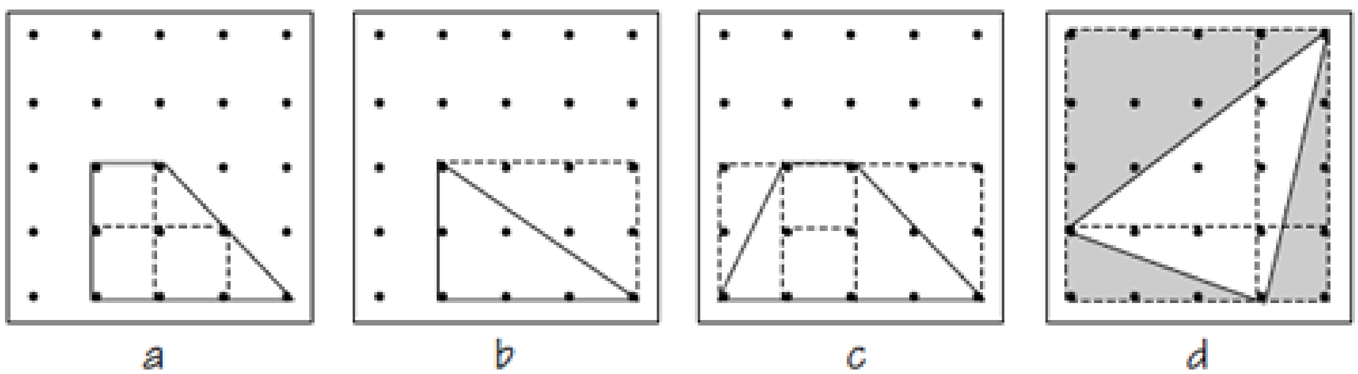

Fig 1: Google images (2016)

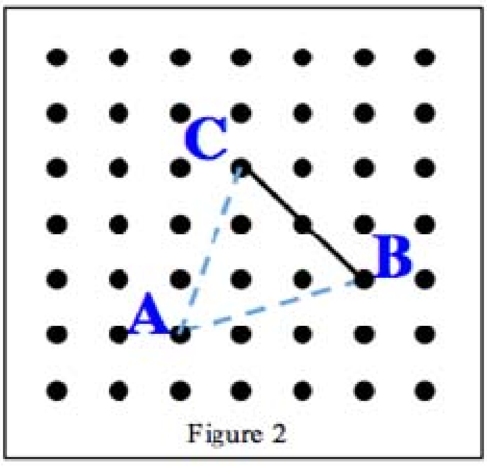

Fig 2: Google images (2016)

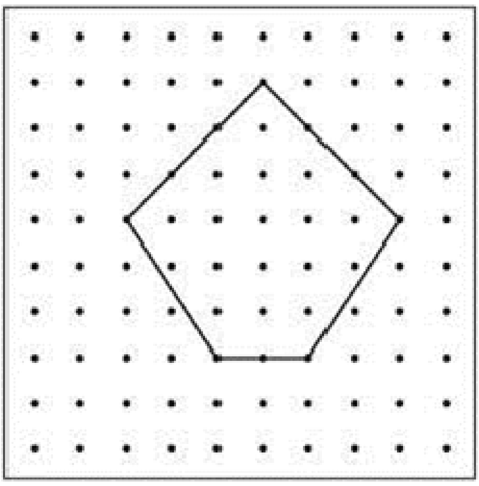

Fig 3: Google (2016)

Using the geo-board is one way to prepare students for occupations in the real world which may require measurements for unusual shapes as such as Carpentry, interior decorating, fashion designing, air traffic controlling and much more. With the geo-board, the students can design almost any of the polygons used in geometry.

The next area that was covered was "Acting it out (making a systematic list) strategy." First, students were shown a video on the subject; the video gave students the opportunity to see visually how systematic lists are planned and constructed. After watching the video, there was a group discussion on how the students felt about what they saw. The lesson was also expatiated by asking students the following questions. Dominique has 45 cents in her pocket which does not include a solid quarter. Can you tell all possible combinations of coins that add up to 45 cents? A table (list) was created, and columns were labeled "Dimes," "Nickels," and "Pennies" to help students answer the questions and we used beans, beads, and balls to represent Dimes, Nickels, and Pennies, respectively. Then we began filling in the rows with combinations of beans, beads and balls that add up to 45 cents. In the first row of the dime column, we put the maximum number of dimes Leslie could have. In the Nickels column, we showed the maximum number of nickels possible with the number of dimes already listed. Also in the pennies column we listed the maximum number of pennies Dominique had to add to his dimes and nickels to make 45 cents.

The next area to be learned was the use of Virtual (electronic) manipulative. The students were enthusiastic about learning and using this form of manipulative. They seemed genuinely excited, and it was very pleasing to me seeing them completely involved with what they were learning. Virtual manipulative is "an interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer, Bolyard & Spikell, 2002, p. 373). With this form of manipulative, students were taught how to slide, flip, and turn (Reflection, Rotation, and Transformation).

Chapter 4
Analysis and results

The students began this nine-week project by completing an eight-question survey. This survey contained questions regarding student interest in mathematics, the way they feel about manipulatives, what they know about manipulatives, their past experiences with manipulatives, and the way they were taught the use of manipulatives in the past. The students filled out the survey anonymously so that they would not feel intimidated to answer truthfully. The purpose of this study is to see the results as a group rather than individuals. This survey will be used as a pre-test for this research project.

To conclude this nine-week project, Students were given a post-test survey and an assessment test to determine how far they came along. This study consisted of the same questions that were in the pre-test survey. Using the same survey gives the researcher the most accurate account of any changes made by the student. It leaves no room for mistake or misunderstanding on the part of the student. Both pre-test and post-test surveys were done so that the researcher could obtain the most accurate results.

Prediction results

Prediction 1: Students' interest in learning mathematics will increase. The results for prediction number one produced the largest positive Increase in student opinion. Question number one of the pre and post survey asked, "How do you feel about mathematics? In the pre-survey, forty percent were interested in mathematics in comparison to the post survey that yielded an increase to eighty-five percent being interested. (Figure1).

There are several facts to take into consideration for the large increase in interest on the part of the students. The actual material (manipulative) presented had to be of some interest to the students. The ability to relate it to their lives helps students' gain a better comprehension of most of the concepts. Working with computers also helped to increase their enthusiasm. As a researcher, watching the students manipulate the manipulatives reinforced the fact that the projects chosen would be of interest to middle school students.

Mathematics Survey

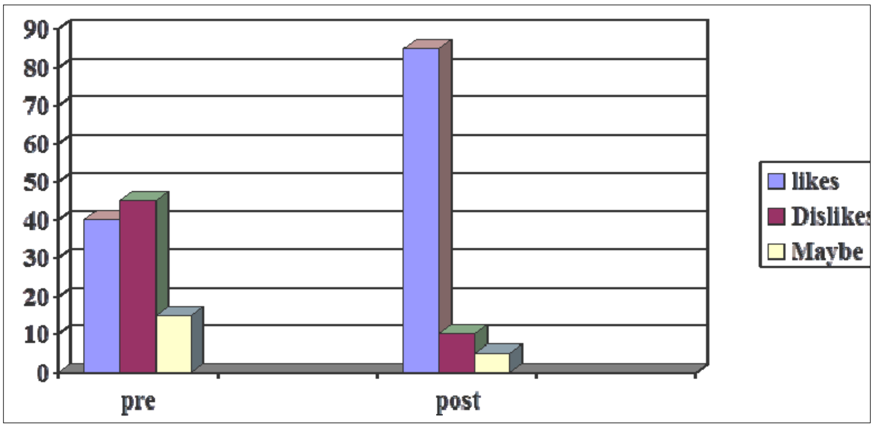

Fig 1: How do you feel about mathematics?

Prediction 2: Students will make a significant improvement in their geometric and algebraic thinking skills. For this prediction, there were two different questions on both pre and post survey. The first question asked, "Do you like taking a mathematics test?" The "Likes" response changed drastically from 34% during pre-test survey to 65% at post-survey (Figure 2). The second question was "Do you like the use of manipulative materials to enhance understanding of mathematics? Again this was the question that yielded low percentage in the pre-survey. However, the percentage that likes the use of manipulative during post survey rose significantly (75%). The high percentage may be due to the facts that many students voiced not knowing what manipulatives are, or had limited knowledge and use of them during the pre-survey section. Therefore, after being exposed to countless different manipulatives during the course of the nine-week, students began to appreciate the power of manipulatives.

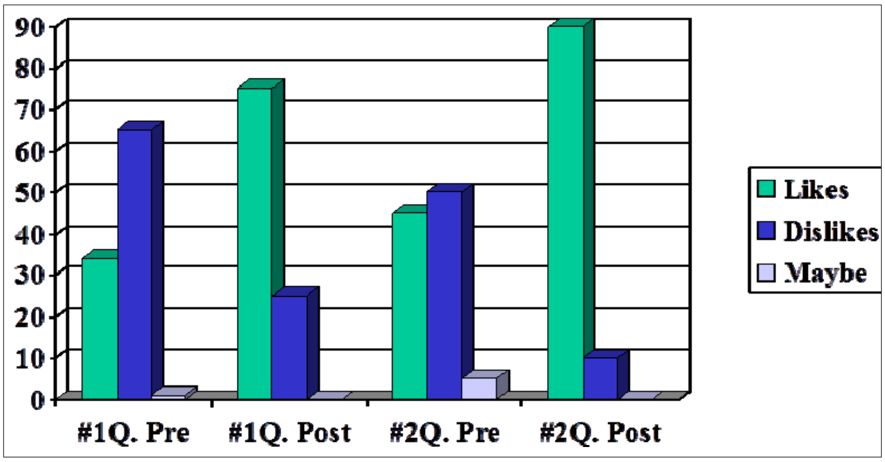

Fig 2: #1question. Do you like taking mathematics test?
#2 question. Do you like the using of manipulative materials to enhance understanding of Mathematics?

Chapter 5

Implications for teaching and learning

With the examination of the research study along with research data, the implications for teaching and learning became apparent. The usefulness of this research project for teaching and learning is addressed in these last pages. Learning in essence, do not only entail acquiring and recalling a transmitted messages; instead research suggests that students learn mathematics better when they construct their individual mathematical understanding. When teachers perceive learning as knowledge construction, they modify their opinions or beliefs on assessment, instruction, curriculum, and develop more powerful approaches for linking those opinions and mathematics and appropriately design better instructional tools. Those instructional tools could be minds-on and hands-on that accord students the opportunity to explore, discover, discuss, and meaningfully create mathematical ideas that involve real world situations that are significant, fascinating as well as engaging.

Results of the study

- **Assumption 1:** Students' interest in learning mathematics will increase.
- **Assumption 2:** Students will make a significant improvement in their geometric and algebraic thinking skills.

The above research hypotheses have been validated. When teachers create an enabling environment where students become active learners through hands-on activity with concrete objects, called manipulatives, they exponentially boost students' comprehension of mathematical concepts as well as their interest in mathematics. This along with other research before it has proven that students with limited exposure to manipulatives have limited knowledge of mathematical concepts, therefore, performs less than those students who are exposed to the use of manipulatives. After participation in problems involving manipulatives, students made a significant improvement in their geometric and algebraic thinking skills. This result agrees with Sue (1996) and Silverman's (1998) conclusion that children who are exposed at an early stage with manipulative materials develops grow up playing with programmable beads, will develop richer intuitions and better geometric understanding than children who did not. Suydam and Higgins (1977), in a review of activity-based mathematics learning in grades K-8, explains that mathematics achievement increases with the use of manipulatives.

Need for continued research

As stated earlier, there is a great need for more documented research in the area of the results of using manipulatives to enhance understanding of mathematics. It has been proven to make a change in students' lives that have gone through a thorough well designed hands-on program. Unfortunately, the results are still in limited quantity. Manipulative is an area of research in need of more exhaustive, in-depth research. This positive documentation will only fuel the need for the financial resources to design programs for all teachers in all schools.

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