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Unsteady MHD boundary layer flow of nanofluid over a stretching sheet with variable viscosity and viscous dissipation

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Abstract
The unsteady magnetohydrodynamics free convection flow of an incompressible viscous nanofluid past a stretching surface is analyzed by taking into an account the variable viscosity and viscous dissipation under the influence of Hall currents effect. The boundary layer equations are governed by coupled nonlinear partial differential equations and transformed into a set of non-linear ordinary differential equations with the help of local similarity transformations. The coupled non-linear ordinary differential equations have been solved numerically by the implicit finite difference method. The effects of flow parameters such as magnetic parameter *M*, Hall parameter *m*, Prandtl number *Pr*, Eckert number *Ec*, thermophoresis parameter *Nt*, Brownian motion parameter *Nb* on axial velocity, transverse velocity, temperature and concentration profiles are investigated analyze through graphs. Physical quantities such as skin friction coefficient $-f''(0)$, heat transfer coefficient $-\theta'(0)$ and mass transfer coefficient $\phi'(0)$ are also computed and are shown in table.

Keywords: MHD, viscous dissipation, nanofluid, implicit finite difference method, variable viscosity, hall parameter

Introduction
The study of the flow of electrically conducting fluid in the presence of magnetic field is important from the technical point of view and such types of problems have received much attention by many researchers. The specific problem selected for study is the flow and heat transfer in an electrically conducting fluid adjacent to the surface. The surface is maintained at a uniform temperature T_w, which may either exceed the ambient temperature T_∞ or may be less then T_∞. When $T_w > T_\infty$, an upward flow is established along the surface due to free convection, where as for $T_w < T_\infty$, there is a down flow. The interaction of the magnetic field and the moving electric charge carried by the flowing fluid induces a force, which tends to oppose the fluid motion. The velocity is very small so that the magnetic force, which is proportional to the magnitude of the longitudinal velocity and acts in the opposite direction, is also very small. Additionally, a magnetic field of strength acts normal to the surface. Consequently, the influence of the magnetic field on the boundary layer is exerted only through induced forces within the boundary layer itself, with no additional effects arising from the free stream pressure gradient. The stress work effects in laminar flat plate natural convection flow have been studied by Ackroyd [1]. However, the influence and importance of viscous stress work effects in laminar flows have been examined by Gebhart [2] and Gebhart and Mollendorf [3]. In both of the investigations, special flows over semi-infinite flat surfaces parallel to the direction of body force were considered.

In all the above-mentioned studies, the viscosity of the fluid was assumed to be constant. However, it is known that the fluid physical properties may change significantly with temperature changes. To accurately predict the flow behavior, it is necessary to take into account this variation of viscosity with temperature. Recently, many researchers investigated the effects of variable properties for fluid viscosity and thermal conductivity on flow and heat transfer over a continuously moving surface. Seddeek [4] investigated the effect of variable viscosity on hydro magnetic flow past a continuously moving porous boundary. Seddeek [5] also studied the effect of radiation and variable viscosity on an MHD free convection flow past a semi-infinite flat plate within an aligned magnetic field in the case of unsteady flow. Dandapat *et al.* [6] analyzed the effects of variable viscosity, variable thermal conducting, and thermocapillarity on the flow and heat transfer in a laminar liquid film on a horizontal stretching sheet. Mukhopadhyay [7] presented solutions for unsteady boundary layer flow and heat transfer over a stretching surface with variable fluid viscosity and thermal diffusivity in presence of wall suction.

When the conducting fluid is an ionized gas and the strength of the applied magnetic field is large, the normal conductivity of the magnetic field is reduced to the free spiraling of electrons and ions about the magnetic lines force before suffering collisions and a current is induced in a normal direction to both electric and magnetic field. This phenomenon is called Hall effect. When the strength of magnetic field is strong, one can’t neglect the effect of Hall current. It is of considerable importance and interest to study how the results of the hydrodynamical problems get modified by the effect of Hall currents. The Hall Effect is due merely to the sideways magnetic force on the drifting free charges. The electric field has to have a component transverse to the direction of the current density to balance this force. In many works on plasma physics, the Hall Effect is ignored. But if the strength of magnetic field is high and the number density of electrons is small, the Hall Effect cannot be disregarded as it has a significant effect on the flow pattern of an ionized gas. Hall Effect results in a development of an additional potential difference between opposite surfaces of a conductor for which a current is induced perpendicular to both the electric and magnetic field. This current is termed as Hall current. The effect of Hall current on the fluid flow with variable concentration has many applications in MHD power generation, in several astrophysical and metrological studies as well as in plasma flow through MHD power generators. Abo-Eldahab *et al.* [8] and Salem and Abd El-Aziz [9] deal with the effect of Hall current on a steady laminar hydromagnetic boundary layer flow of an electrically conducting and heat generating/absorbing fluid along a stretching sheet. Abd El-Aziz [10] investigated the effect of Hall currents on the flow and heat transfer of an electrically conducting fluid over an unsteady stretching surface in the presence of a strong magnet. S.Shateyi [11] studied the variable viscosity on magnetic hydrodynamic fluid flow and heat transfer over an unsteady stretching surface with Hall effects. Srinivas Maripala and Kishan Naikoti [12], studied the unsteady MHD free convection flow of an incompressible viscous fluid past a stretching surface with variable viscosity and viscous dissipation.

Nanofluids are fluids that contain small volumetric quantities of nanometer-sized particles, called nanoparticles. The nanoparticles used in nanofluids are typically made of metals (Al, Cu, Ag, Au, Fe), oxides (Al2O3, CuO, TiO2), metal carbides (SiC), nonmetals(graphite carbon nanotubes), nitrides (AlN, SiN) and others. Common base fluids include water, ethylene glycol and oil. Nanofluids have enhanced thermophysical properties such as thermal conductivity, thermal diffusivity, viscosity and convective heat transfer coefficients compared to those of base fluids. Moreover, the presence of the nanoparticles enhance the electrical conductivity property of the nanofluids, hence are more susceptible to the influence of magnetic field than the conventional base fluids. The suspended metallic or nonmetallic nanoparticles change the transport properties and heat transfer characteristics of the base fluid, hence enhance the heat transfer of the base fluid. Typical thermal conductivity enhancements are in the range of 15-40% over the base fluid and heat transfer coefficient enhancements have been found up to 40%. It seems that Choi [13] was the first to call the mixture of the base fluids with solid nanoparticle as nanofluid. More recently, the study of conductive heat transfer in nanofluids have achieved great success in various industrial applications a large number of experimental and theoretical have been carried out by numerous researchers on thermal conductivity of nanofluids with different physical situations [16, 22].

In the present investigation, it is proposed to study the effect of magnetohydrodynamics unsteady free convection flow of an incompressible viscous Nanofluid past a stretching surface is analysis by taking into account the Hall effects and viscous dissipation. Fluid viscosity is assumed to vary as an exponential function of temperature while the fluid thermal diffusivity is assumed to vary as a linear function of temperature. The governing equations are solved by using the implicit finite difference scheme using C-programming.

Mathematical Analysis

We consider the unsteady flow and heat transfer of a viscous, incompressible, and electrically conducting nanofluid past a semi-infinite stretching sheet coinciding with the plane $y = 0$, then the fluid is occupied above the sheet $y \geq 0$. The positive x coordinate is measured along the stretching sheet in the direction of motion, and the positive y coordinate is measured normally to the sheet in the outward direction toward the fluid. The leading edge of the stretching sheet is taken as coincident with z-axis. The continuous sheet moves in its own plane with velocity $U_w(x,t)$ and the temperature $T_w(x,t)$ distribution varies both along the sheet and time. A strong uniform magnetic field is applied normally to the surface causing a resistive force in the x-direction. The stretching surface is maintained at a constant temperature and with significant Hall currents. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected. The effect of Hall current gives rise to a force in the z-direction, which induces a cross flow in that direction, and hence the flow becomes three dimensional. To simplify the problem, we assume that there is no variation of flow quantities in z-direction. This assumption is considered to be valid if the surface is of infinite extent in the z-direction. Further, it is assumed that the Joule heating and viscous dissipation are neglected in this study. Finally, we assume that the fluid viscosity is to vary with temperature while other fluid properties are assumed to be constant. Using boundary layer approximations, the governing equations for unsteady laminar boundary layer flows are written as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\sigma B^2}{\rho(1+m^2)}(u+mw) \quad (2)$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right)+\frac{\sigma B^2}{\rho(1+m^2)}(mu-w) \quad (3)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\mu\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right]+\tau\left\{D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right\}-\frac{1}{\rho_f C_p}\frac{\partial q_r}{\partial y} \quad (4)$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2} \quad (5)$$

The boundary conditions are defined as follows ;

$$u=U_w\,(x,t),\ v=0,\ w=0,\ T=T_w\,(x,t),\ C=C_w\,(x,t)\ \text{at}\ y=0,$$
$$u\to 0, w\to 0, T\to T_w, C\to C_w\ \text{as}\ y\ tends\ to\ \infty, \quad (6)$$

where *u* and *v* are the velocity components along the *x*- and *y*-axis, respectively, *w* is the velocity component in the *z* direction, ρ is the fluid density, β is the coefficient of thermal expansion, μ is the kinematic viscosity, *g* is the acceleration due to gravity, c_p is the specific heat at constant pressure, and *k* is the temperature-dependent thermal conductivity. ρ_f is the density of the base fluid, α_m is the thermal diffusivity, D_B is the Brownian diffusion coefficient, D_T is the thermophoresis diffusion coefficient and C_ρ is the specific heat at constant pressure. Here τ is the ratio of the effective heat capacity of the nanoparticle material and the heat capacity of the ordinary fluid, T is the fluid temperature and C is the nanoparticle volume fraction. Following Elbashbeshy and Bazid [14], we assume that the stretching velocity $U_w\,(x,t)$ is to be of the following form

$$U_w=\frac{bx}{(1-ct)}, \quad (7)$$

where *b* and *c* are positive constants with dimension reciprocal time. Here, *b* is the initial stretching rate, whereas the effective stretching rate *b/*(1 – *ct)* is increasing with time. In the context of polymer extrusion, the material properties and in particular the elasticity of the extruded sheet vary with time even though the sheet is being pulled by a constant force. With unsteady stretching, however, c^{-1} becomes the representative time scale of the resulting unsteady boundary layer problem. Using Rosseland's approximation [15], for radiation we can write

$$q_r=-\frac{4\sigma_1}{3K_1}\frac{\partial T^4}{\partial y}, \quad (8)$$

where σ_1 is the Stefan–Boltzmann constant and K_1 is the mean absorption coefficient. Assuming the temperature difference within the flow is such that T^4 may be expanded in a Taylor series about T_∞ and neglecting higher order terms we get $T^4\approx 4T_\infty^3T-3T_\infty^4$. Hence from Eq. (8),
using the above result,

we have $$\frac{\partial q_r}{\partial y}=-\frac{16\sigma_1 T_\infty^3}{3K_1}\frac{\partial^2 T}{\partial y^2} \quad (9)$$

The surface temperature T_W of the stretching sheet varies with the distance *x* along the sheet and time *t* in the following form:

$$T_w\,(x,t)=T_\infty+T_0\left[\frac{bx^2}{v^*}\right](1-\alpha t)^{-3/2}, \quad (10)$$

where T_0 is a (positive or negative; heating or cooling) reference temperature. The governing differential equations (1) – (5) together with the boundary conditions (6) are non dimensiona -lized and reduced to a system of ordinary differential equations using the following dimensionless variables:

$$\eta=\left(\frac{b}{v}\right)^{\frac{1}{2}}((1-\alpha t))^{-1/2}y,\ \psi=(vb)^{\frac{1}{2}}((1-\alpha t))^{-\frac{1}{2}}xf(\eta),\ w=bx((1-\alpha t))^{-1}h(\eta),$$

$$T = T_\infty + T_0\left[\frac{bx^2}{v}\right](1-\alpha t)^{-3/2}\theta(\eta),\ B^2 = B_0{}^2((1-ct))^{-1},\ Ec = \frac{u^2}{C_w(T_w-T_\infty)},\ \varphi(\eta) = \frac{C-C_\infty}{C_w-C_\infty} \tag{11}$$

where ψ(x, y,) is the physical stream function which automatically assures mass conservation (1) and B_0 is constant. We assume the fluid viscosity to vary as an exponential function of temperature in the non dimensional form $\mu = \mu_\infty e^{-\beta_1\theta}$, where μ_∞ is the constant value of the coefficient of viscosity far away from the sheet, β_1 is the variable viscosity parameter. The variation of thermal diffusivity with the dimensionless temperature is written as $k = k_0(1+\beta_1\theta)$ where β_2 is a parameter which depends on the nature of the fluid, *k*0 is the value of thermal diffusivity at the temperature T_w.
Upon substituting the above transformations into (1)–(5) we obtain he following:

$$f''' - \beta_1\theta' f'' + e^{\beta_1\theta}\left[f f'' - (f')^2 - S\left(f' + \frac{\eta}{2}f''\right) - \frac{M^2}{1+m^2}(f' + mh)\right] = 0 \tag{12}$$

$$h'' - \beta_1\theta' h' + e^{\beta_1\theta}\left[f h' - h f' - S\left(h + \frac{\eta}{2}h'\right) + \frac{M^2}{1+m^2}(mf' - h)\right] = 0 \tag{13}$$

$$(1+\beta_2\theta)\theta'' + \beta_2(\theta')^2 + \Pr(f\theta' - 2f'\theta) - S(3\theta + \eta\theta') + \Pr(Nb\theta'\varphi' + Nt\,\theta'^2) = 0 \tag{14}$$

$$\varphi'' + Le\, f\varphi' + \frac{Nt}{Nb}\theta'' = 0 \tag{15}$$

where the primes denote differentiation with respect to *η*, and the boundary conditions are reduced to

$$f(0) = 0,\ f'(0) = 1,\ h(0) = 0,\ \theta(0) = 1,\ \varphi(0) = 1 \qquad 16(a)$$

$$h(\infty) = 0,\ f(\infty) = 0,\ \theta(\infty) = 0,\ \varphi(\infty) = 0 \qquad 16(b)$$

The physical quantities of interest are the skin friction coefficient C_f, the local Nusselt number Nu_x and the local Sherwood number Sh_x which are defined as

$$C_f = \frac{\tau_w}{\rho_f u_w^2},\ Nu_x = \frac{xq_w}{k(T_w - T_\infty)},\ Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)}, \tag{17}$$

where τ_w is the shear stress at the stretching surface, q_w and q_m are the wall heat and mass fluxes, respectively.
Hence using Eq. (12) we get

$$Re_x^{1/2}C_f = f''(0),\ Nu_x\, Re_x^{-1/2} = -\theta'(0),\ Sh_x\, Re_x^{-1/2} = -\varphi'(0) \tag{18}$$

where $Re_x = u_w(x,t)x/v$ is the local Reynolds number based on the stretching velocity $u_w(x,t)$.

The governing non dimensional equations (12) – (15) along with the boundary conditions 16(a)-16(b) are solved by the implicit finite difference scheme using C-programming.

Table 1: Effects of magnetic parameter *M*, Hall parameter *m*, Brownian motion parameter *Nb* and thermophoresis parameter *Nt* on skin friction coefficient $-f''(0)$, heat transfer coefficient $-\theta'(0)$ and mass transfer coefficient $-\phi'(0)$

		$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
M	1.0	*β1=0.1,β2=0.2,S=1.0,Pr=0.72,m=5.0,Ec=0.1,Le=1.0,Nb=0.1,Nt=0.1*		
	3.0	1.52693	1.32081	0.98341
	5.0	2.80258	1.20452	0.86121
		4.39612	1.02935	0.82342

m	1.0 3.0 5.0	*β1=0.1,β2=0.2,S=1.0,Pr=0.72,M =1.0,Ec=0.1,Le=1.0,Nb=0.1,Nt=0.1*		
		1.80210 1.39258 1.20548	1.20802 1.23212 1.29141	0.890341 0.92341 0.98432
Nb	0.1 3.0 5.0	*β1=0.1,β2=0.2,S=1.0,Pr=0.72,M=1.0,m=5.0,Ec=0.1,Le=1.0,Nt=0.1*		
		0.52103 0.38193 0.29123	0.94524 0.53162 0.12134	2.12945 2.58382 2.69341
Nt	0.1 3.0 5.0	*β1=0.1,β2=0.2,S=1.0,Pr=0.72,M=1.0,m=5.0,Ec=0.1,Le=1.0,Nb=0.1*		
		0.51224 0.46192 0.42321	0.94523 0.52333 0.39243	2.24294 2.53212 2.64315

Results and Discussion

In order to solve the unsteady, non-linear coupled differential equations (12) – (15) along with boundary conditions (16) an implicit finite difference scheme of Cranck-Nicklson type has been employed. The system equations are reduced to tri-diagonal system of equations which are solved by Thomas algorithm. The η_{max} chosen as 10 corresponds to $\eta \to \infty$ after some preliminary investigation so that the last two boundary condations16 (a), 16(b) is satisfied at $\eta \to \infty$, within the tolerance limit of 10^{-5} the mesh size has been fixed as 0.01. The numerical computation are have been carried out for the different governing parameters such as magnetic parameter *M*, Hall parameter *m*, Prandtl number *Pr*, Eckert number *Ec*, unsteadiness parameter *S*, viscosity parameter β_1 and diffusivity parameter β_2 only selective figures have been shown here for brevity. Physical quantities such as skin friction coefficient $-f''(0)$, heat transfer coefficient $-\theta'(0)$ and mass transfer coefficient $-\varphi'(0)$ are also computed and are shown in table-I. It is evident that with the increase of magnetic parameter *M*, increases the skin friction parameter values and decrease the heat and mass transfer coefficient values. Hall parameter *m*, decrease the skin friction coefficient values and mass transfer values at the same time increase the heat transfer coefficient values. Brownian motion parameter *Nb*, decrease the skin friction and heat transfer values. Skin friction and heat transfer values decrease and mass transfer values are increased with the increase of thermophoresis parameter *Nt*.

Figures 1(a)-(d) depict the influence of magnetic field parameter *M* on axial velocity, transitive velocity, temperature, and concentration profiles, respectively. Figure 1(a), it is noticed that an increase in the magnetic parameter *M* leads to decrease in the axial velocity, while it can be seen that the temperature profiles increase with the increase of magnetic parameter *M*. It is obvious that the effect of the magnetic parameter results in a decreasing velocity distribution across the boundary layer. This is due to act that the effect of a transverse magnetic field gives rise to a resistive type force called the Lorentz force. This force has a tendency to slow down the motion of the fluid. Figure 1(b), it can be noticed that the effect of magnetic flied increases the transverse velocity filed and reaches to a peak value near the vicinity of the boundary layer and approaches to zero. It is also noticed that more influence of magnetic field will reach peak value and reaches to zero near the boundary layer, whereas the less magnetic field influence have less peak value and it will reach to zero far away from the boundary. Figure 1(d) explains that the concentration profiles are increased with the increase of magnetic parameter *M*.

Figures 2(a) - (d) extract graphical information about the effects of Hall parameter *m*. Figure 2(a) shows that the effect of Hall parameter *m* on axial velocity profiles. It can be seen that the axial velocity increases when Hall parameter m increases. From figure 2(b), it is observed the Hall parameter m increasing from 0 to 1.5 causes the transverse flow increases in η direction. From figures 2(c) and 2(d), it is observed that temperature and concentration profiles decrease with the increase of Hall parameter *m*.This is due to fact that for large values of m the term $\sigma/(1+m^2)$ is very small ; and hence the resistive effect of the magnetic field is to diminished. The effects due to viscous dissipation *Ec* on axial velocity, transverse velocity, temperature, and concentration profiles are plotted and presented in figures 3(a)-3(c). From these figures revels that the influence of viscous dissipation effects is to increase the axial velocity and temperature profiles. It can be observed, from figure 3(b) that the transverse velocity decreases with the increase of Eckert number *Ec*, nearest vicinity of the wall and the reverse phenomenon is observed away from the wall and the effect is high away from the plate. This is due to the fact that the heat energy is stored in the fluid due to the frictional heating. So we can say that the strong frictional heating slow down the cooling processes and in this case the study suggest that the rapidly cooling of the surface can be made possible if the viscous dissipation can be made as small as possible.

Figure 4(a) -(c) illustrates the influence of variable viscosity β_1 on axial velocity, transverse velocity and temperature profiles, respectively. It is observed that the effective of the variable viscosity $\beta_1$is to reduce the axial velocity profiles, due to increase of β_1 the boundary layer thickness decreases. It can be seen from the figure 4(b) with the increase of variable viscosityβ_1, the transverse velocity increases to a peak value near the boundary wall and then decays rapidly to the relevant free stream velocity. It can be conclude from figure 4(c) that the distribution of temperature θ(η) increases with the increasing values of variable viscosity β_1. Figures 5(a) to 5(c) illustrate the effect of the unsteadiness parameter *S* on axial velocity, transverse velocity and temperature profiles, respectively. From these figures, it can be seen that the increase of unsteadiness parameter *S* is to decrease the transverse magnetic field as well as temperature profiles. It is also observed from Figure 5(b) that the transverse velocity profiles are to decrease greatly near the plate and the reverse happen far away from the plate. Figure 6(a) presents typical profiles of transverse velocity for the different values of the thermal diffusivity parameter β_2. It can be noticed that an increase in thermal diffusivity parameter β_2 leads to increasses transverse velocity profiles. The effect of thermal diffusivity parameter β_2 is to increase the temperature distribution is noticed from Figure 6(b). This is due to the thicking of the thermal boundary layer as a result of increasing of thermal diffusivity. Figures 7(a) and 7(b) explain the effect of thermothoresis parameter *Nt* on temperature and concentration profiles, respectively. It is observed that the temperature field decreases and the concentration profiles are increases with the increase of thermophoresis parameter *Nt.* Figure 8(a) and 8(b) depict that the Brownian motion parameter *Nb*, increases the temperature profiles and decrease the concentration profiles when it increases. Figure 9 presents the Lewis number *Le* effects on concentration profiles, increasing the value of *Le* has the tendency decreases the fluid concentration in the boundary layer as were as the thermal boundary layer thickness.

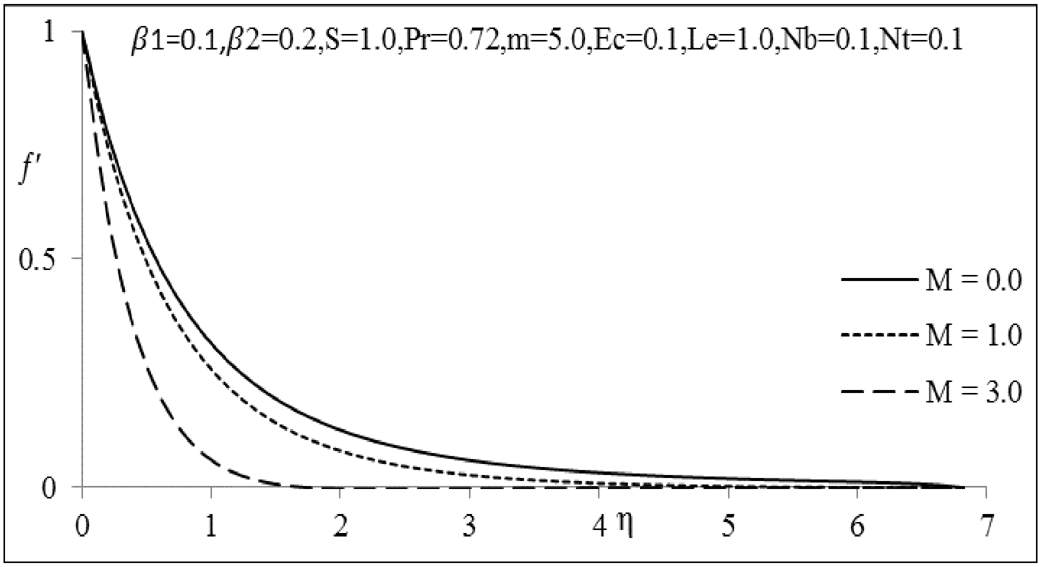

Fig 1(a): The varation axial velocity porfiles with increasing valuses of M

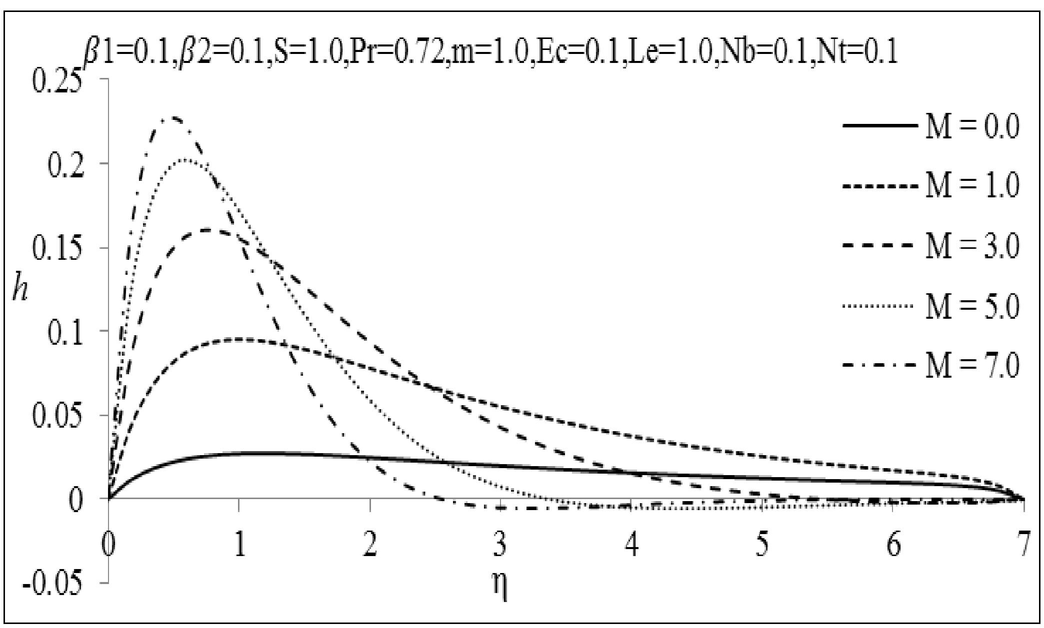

Fig 1(b): Transverse velocity profiles for various values of M

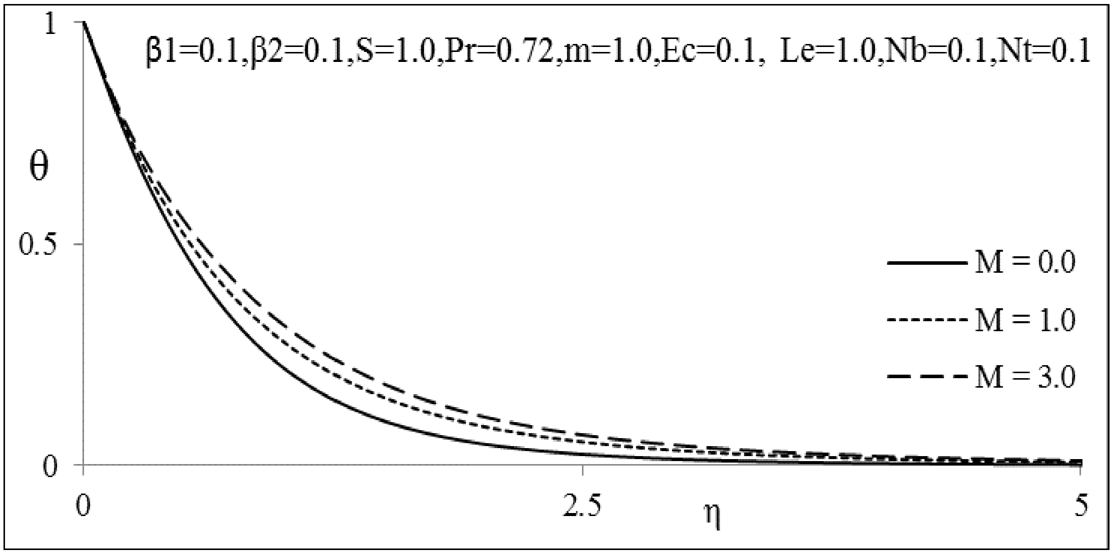

Fig 1(c): Temperature profiles for various values of M

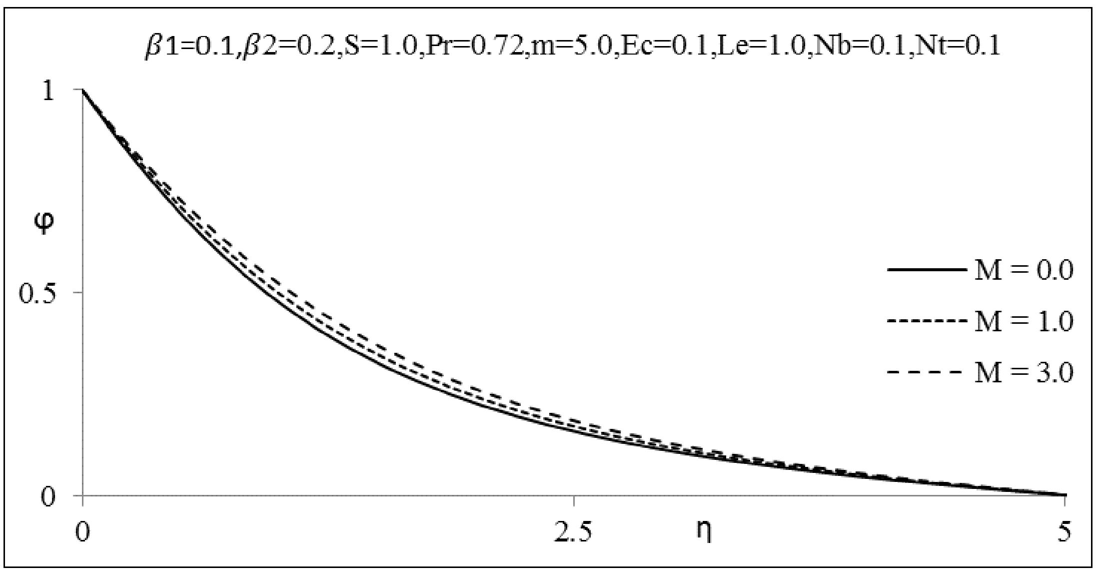

Fig 1(d): Concentration profiles for various values of M

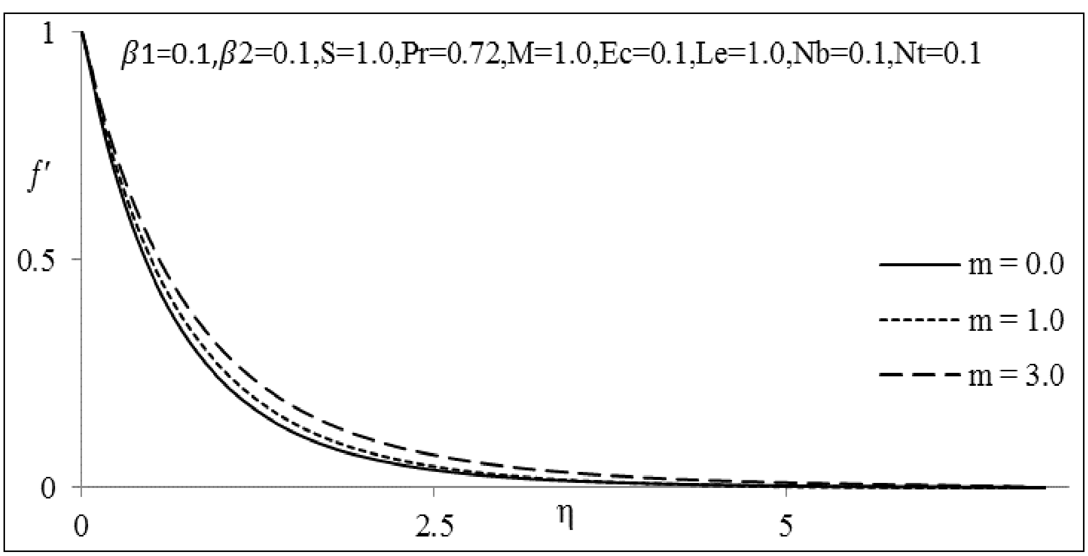

Fig 2(a): The varation axial velocity profiles with increasing values of m

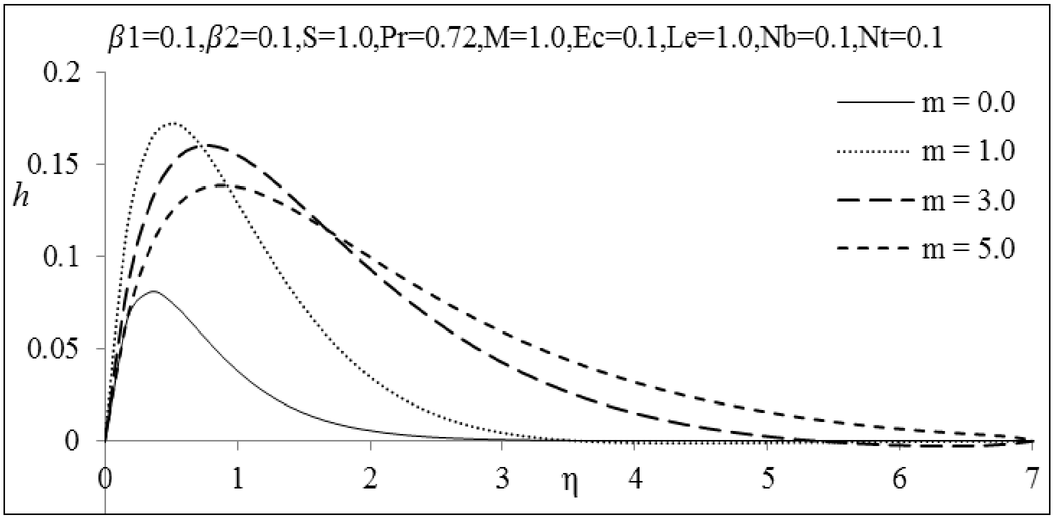

Fig 2(b): Transverse velocity profile for various values of m

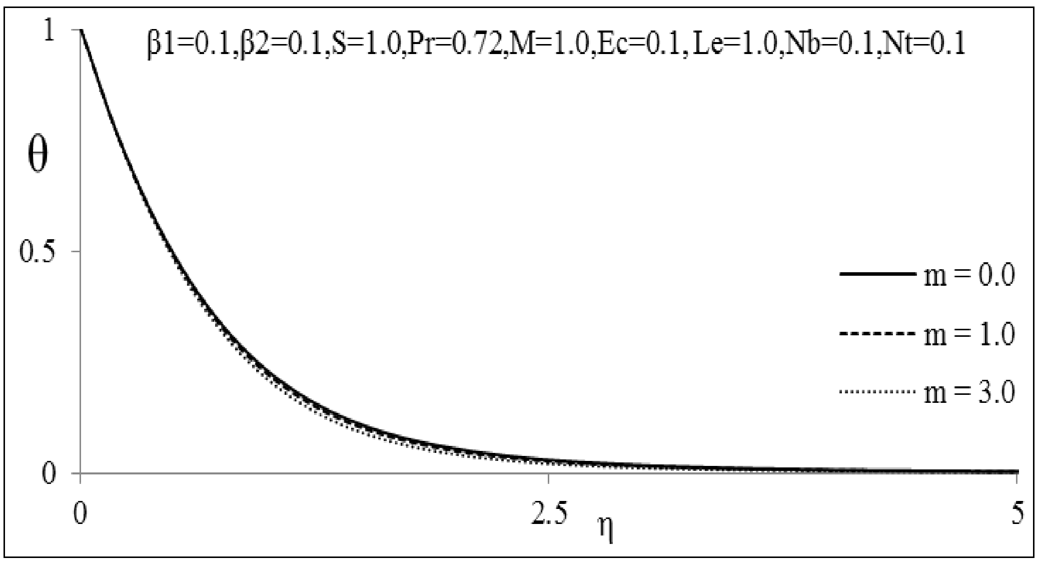

Fig 2(c): Temperature profiles for various values of m

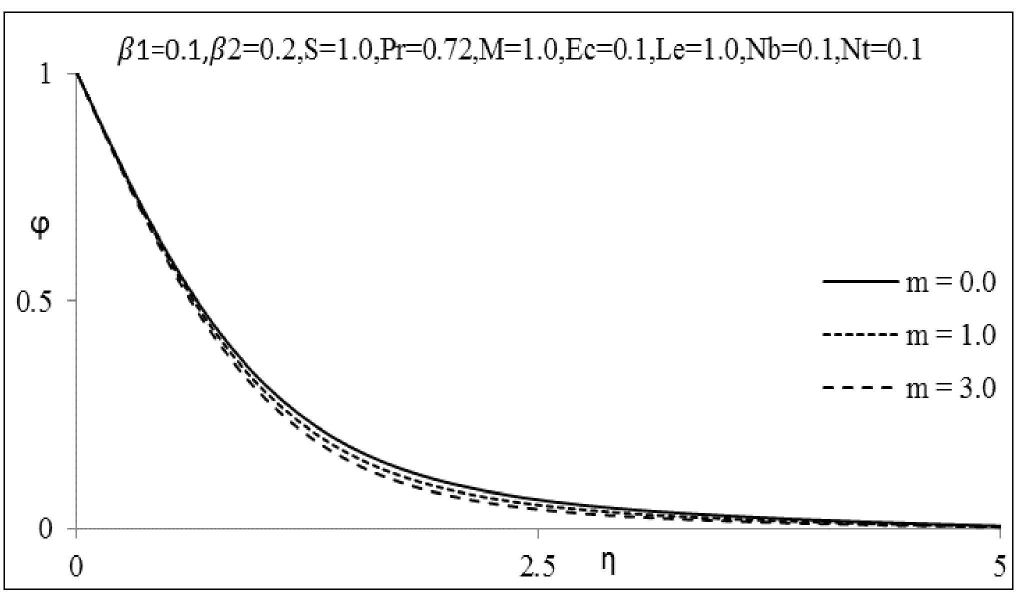

Fig 2(d): Concentration profiles for various values of m

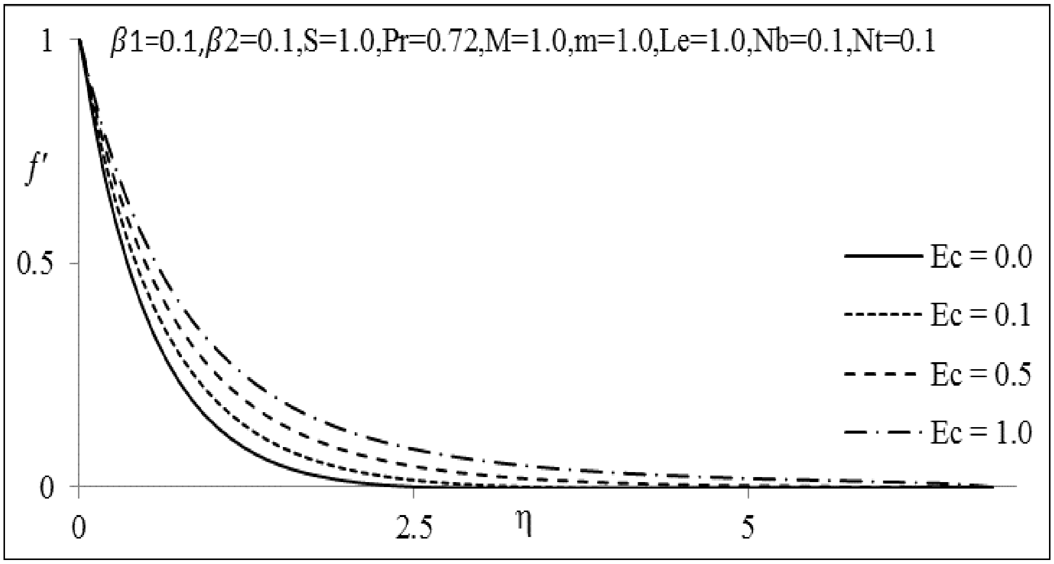

Fig 3(a): The varation axial velocity profiles with increasing values of Ec

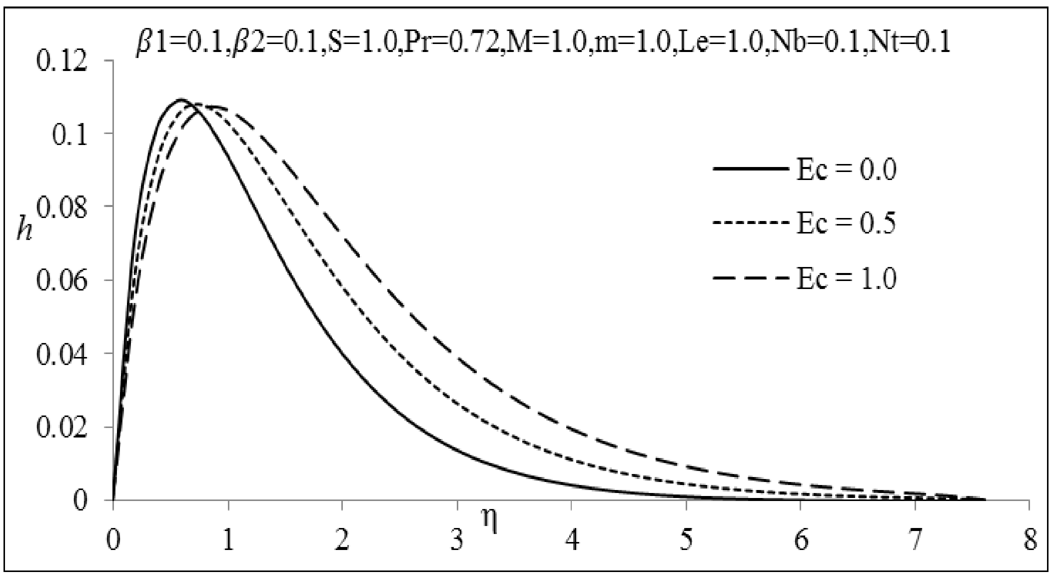

Fig 3(b): Transverse velocity profiles for various values of Ec

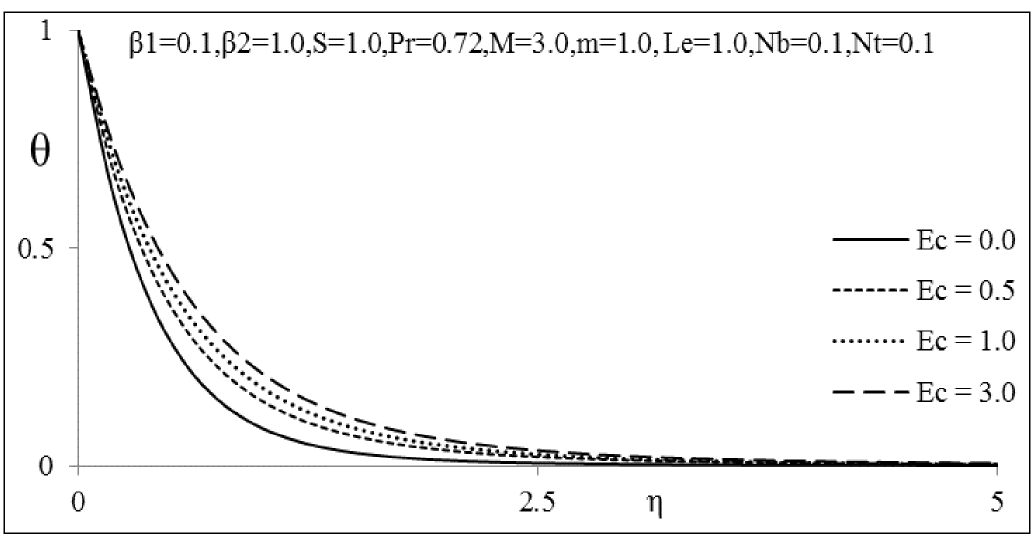

Fig (c): Temperature profiles for various values of Ec

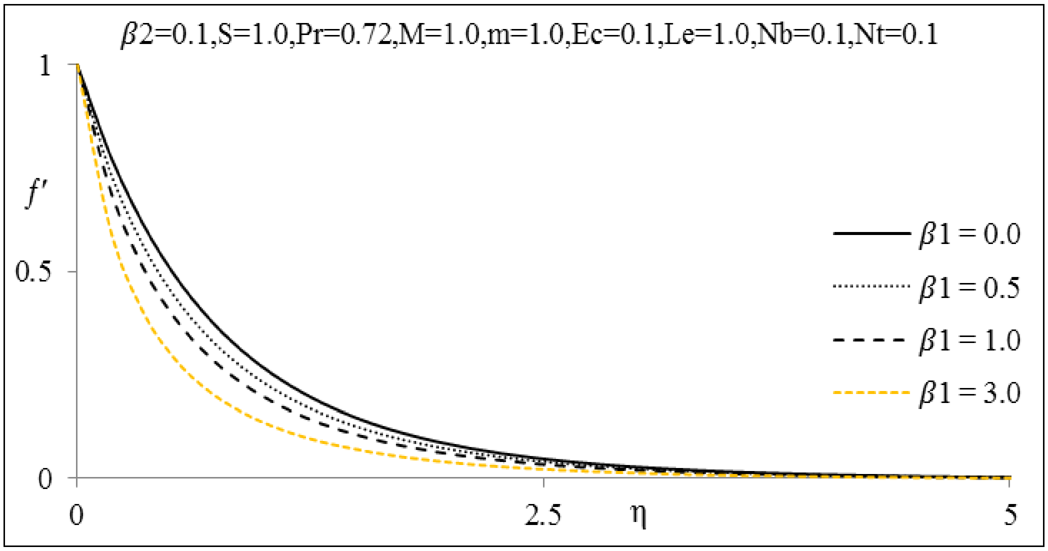

Fig 4(a): The varation axial velocity profiles with increasing values of β1

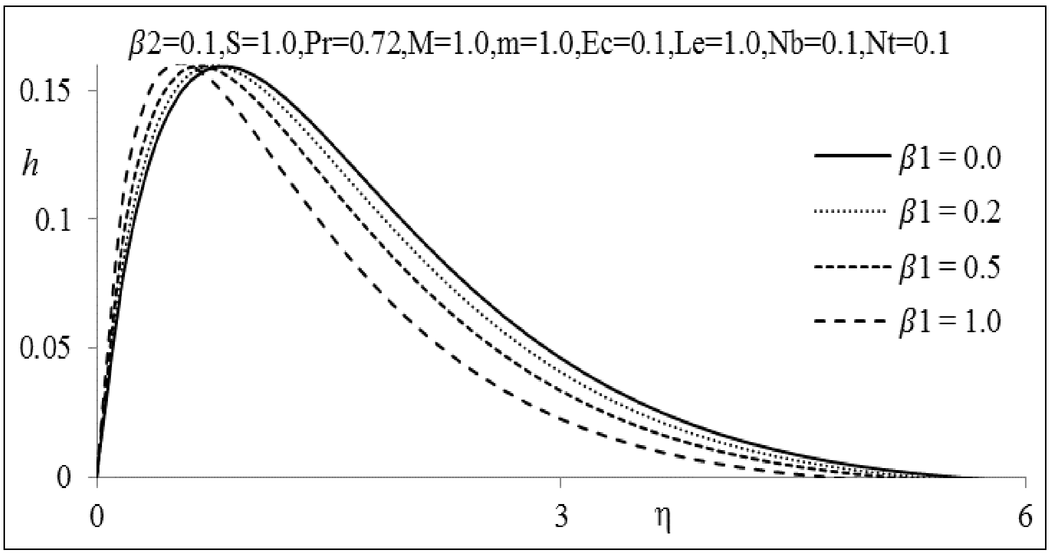

Fig 4(b): Transverse velocity profiles for various values of β1

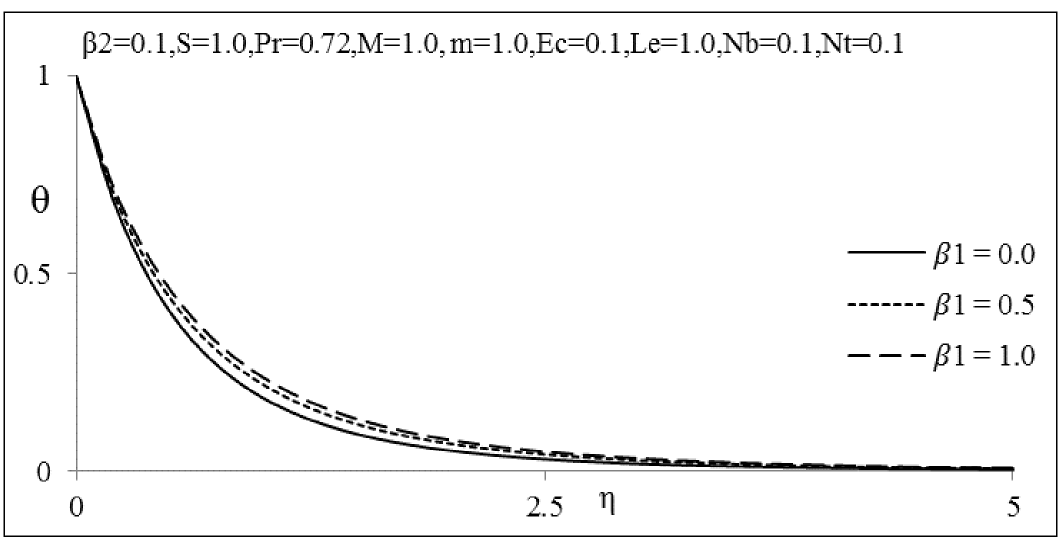

Fig 4(c): Temperature profiles for various values of β1

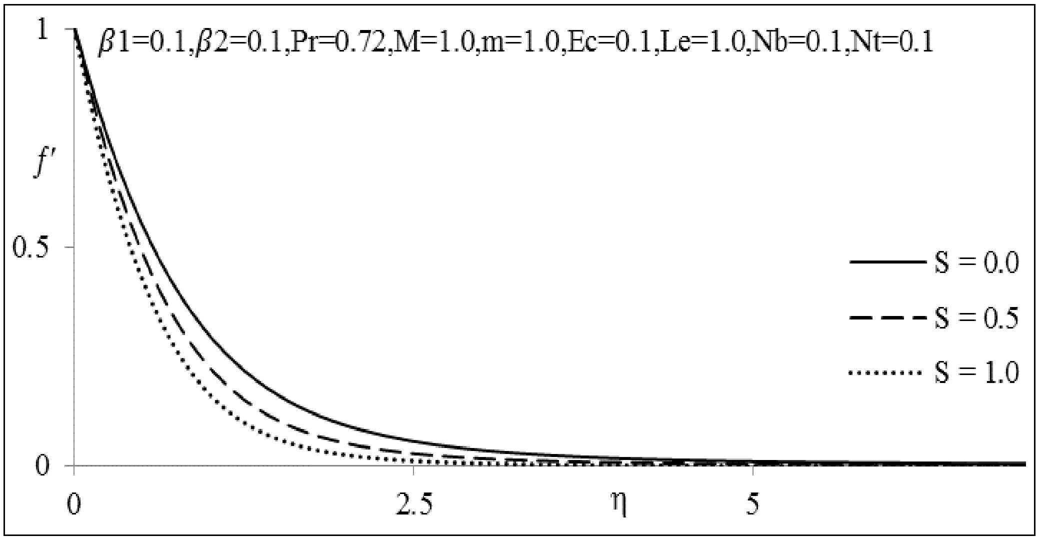

Fig 5(a): The varation axial velocity profiles with increasing values of S

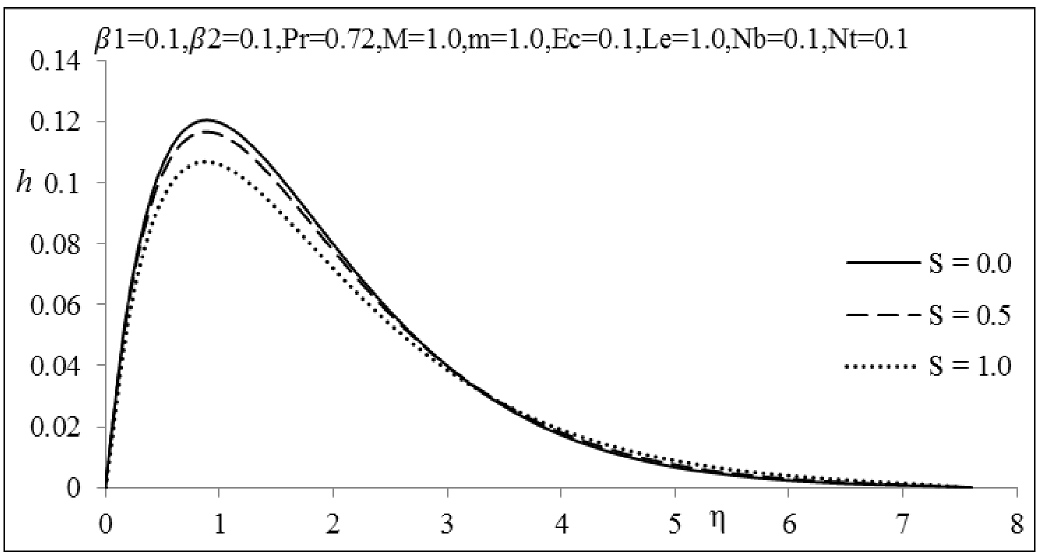

Fig 5(b): Transverse velocity profiles for various values of S

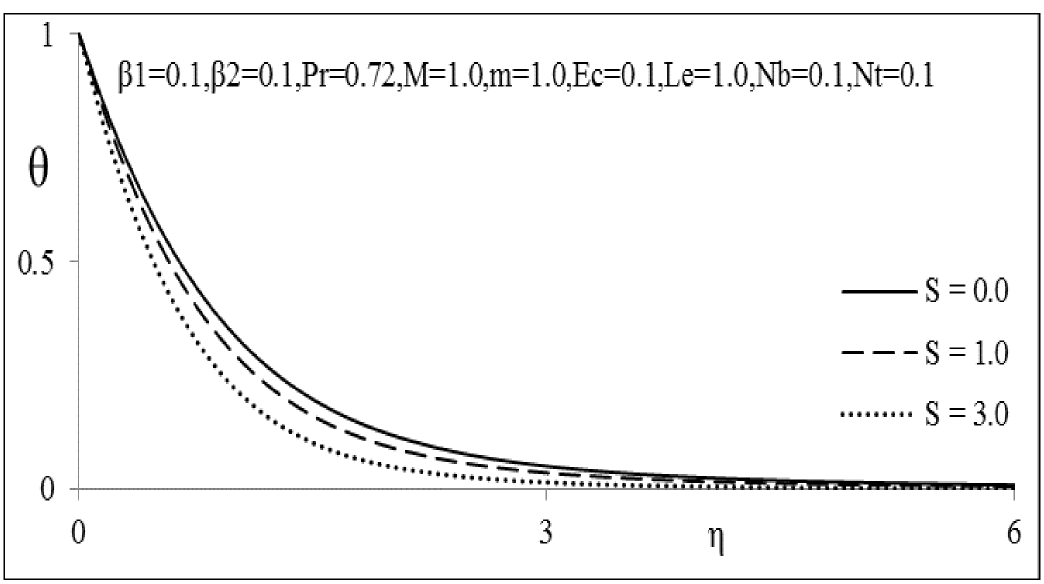

Fig 5(c): Temperature profiles for various values of S

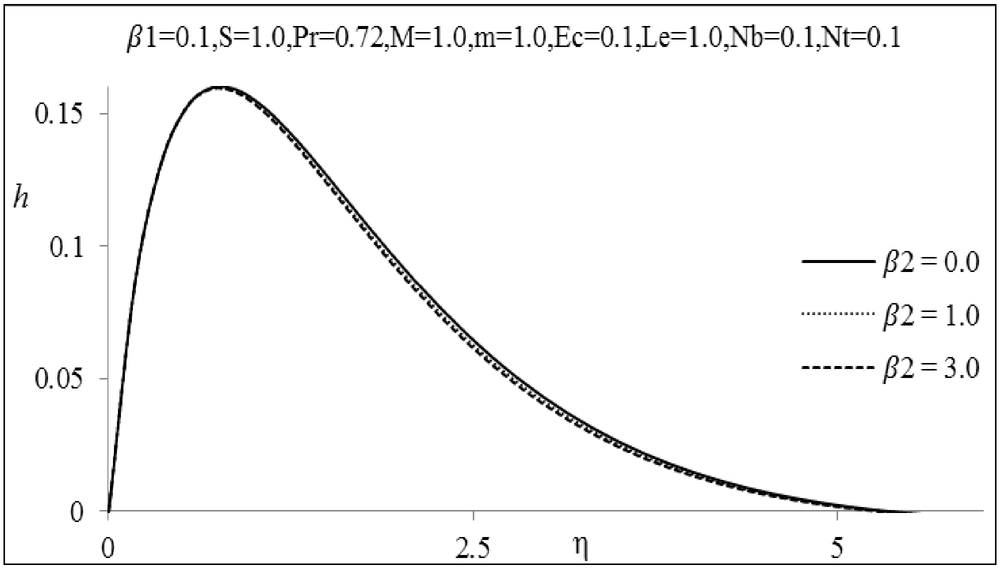

Fig 6(a): Transvers velocity profiles for various values of β2

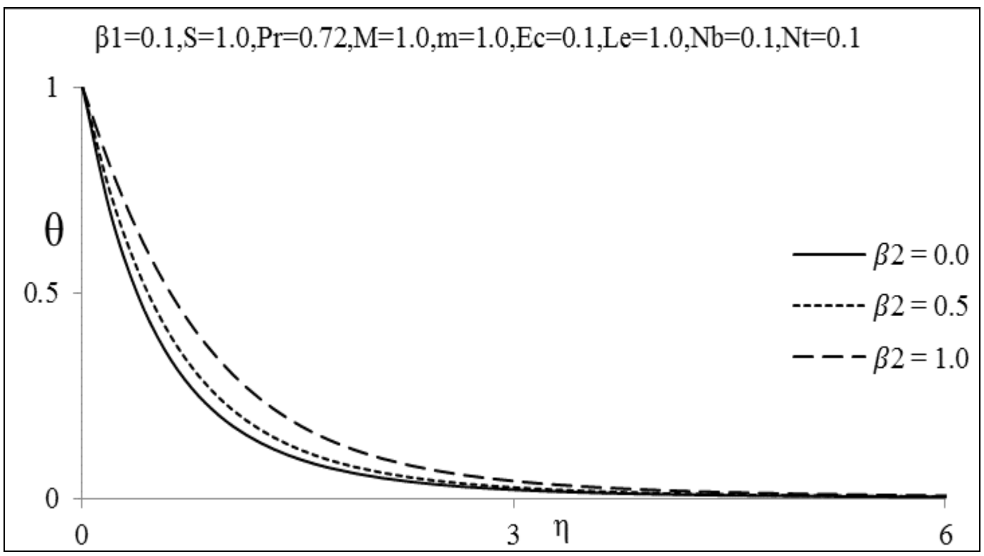

Fig 6(b): Temperature profiles for various values of β2

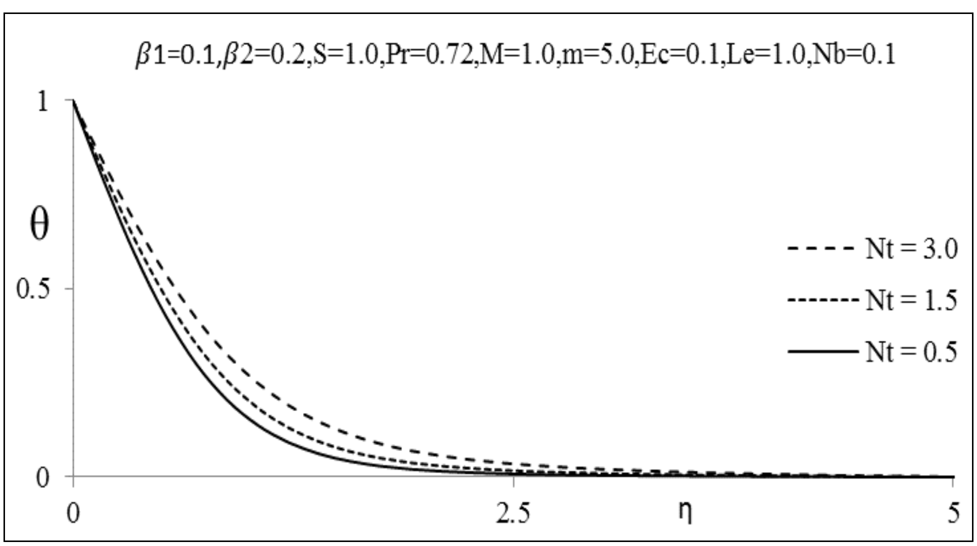

Fig 7(a): Temperature profile for various values of Nt

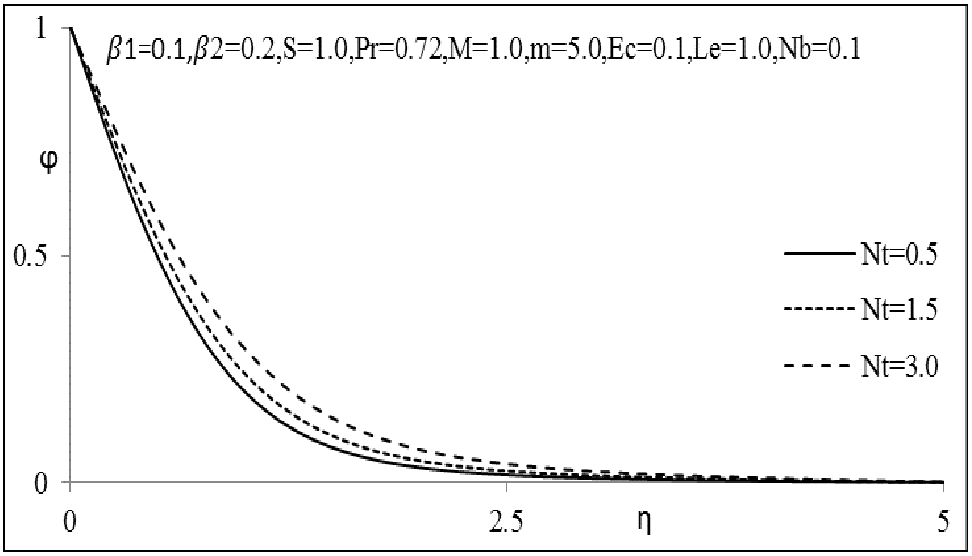

Fig 7(b): Concentration profiles for various values of Nt

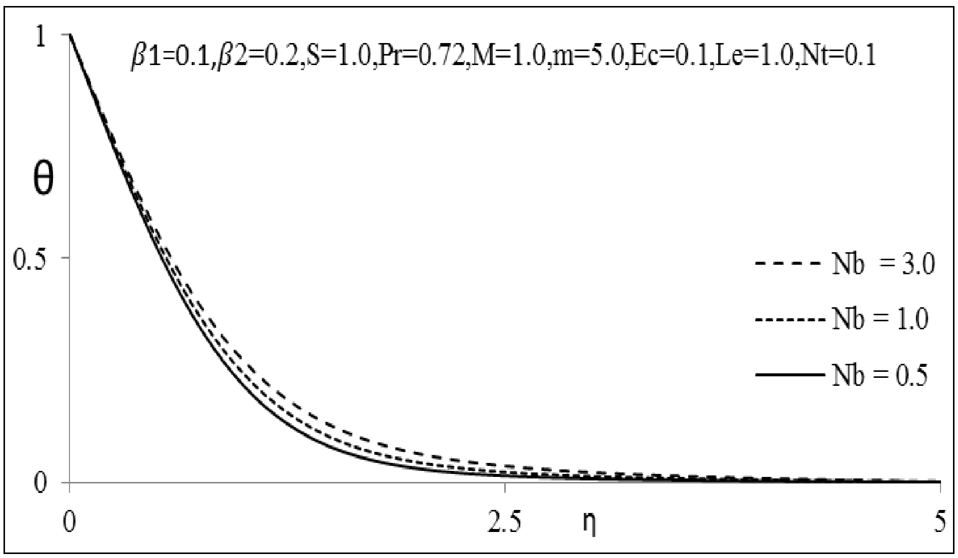

Fig 8(a): Temperature profiles for various values of Nb

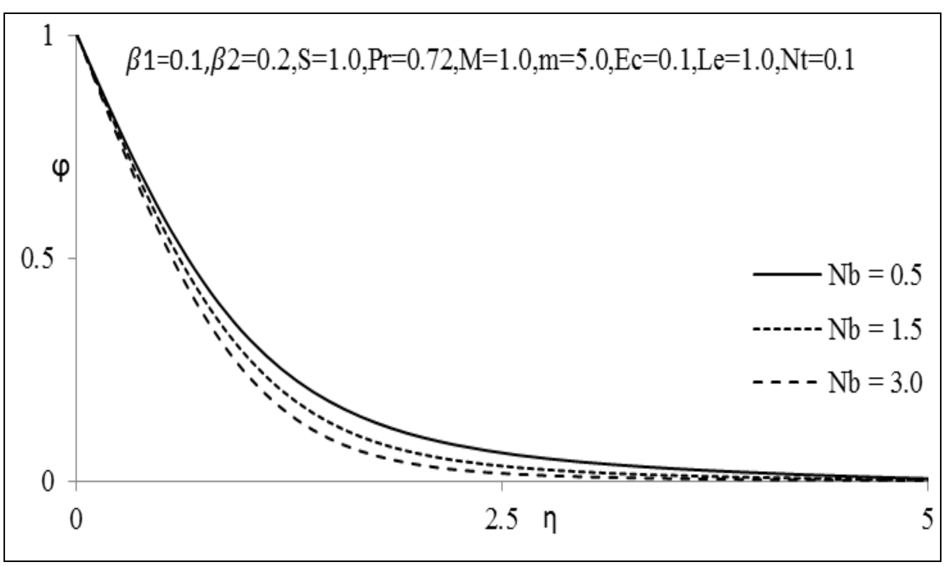

Fig 8(b): Concentration profiles for various values of Nb

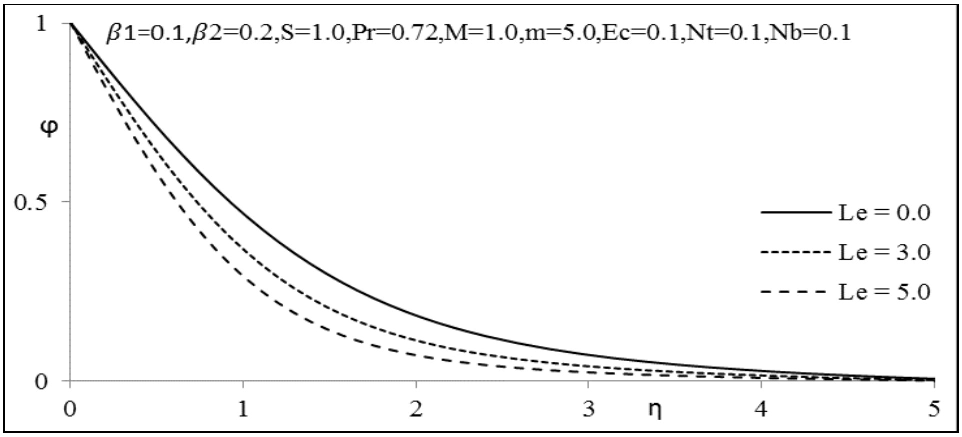

Fig 9: Concentration profiles for various values of Le

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