International Journal of Advanced Research and Development
ISSN: 2455-4030
Impact Factor: RJIF 5.24
www.advancedjournal.com
Volume 2; Issue 5; September 2017; Page No. 622-625

A new integral operator using generalized hypergeometric operator

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Abstract
In this paper, we define a new integral operator using generalized hypergeometric operator for certain p-valent functions in the unit disc. By using new integral operator we obtain many known integral results.

Keywords*:* p-valent functions, generalized hypergeometric functions

1. Introduction

Let A_p denote the class of functions of the form

$$f(z) = z^p + \sum_{k=p+n}^{\infty} a_k z^k \quad , \quad (p, n \in \mathbb{N} = \{1,2, \ldots .\}) \tag{1}$$

Which are analytic and p-valent in the punchered open unit disc $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$.
For functions $f \in A_p$ given by (1) and $g \in A_p$ given by

$$g(z) = z^p + \sum_{k=p+n}^{\infty} b_k z^k,$$

The Hadamard product (or convolution) of f and g is defined by

$$(f * g)(z) = z^p + \sum_{k=p+n}^{\infty} a_k b_k z^k = (g * f)(z).$$

For positive real numbers $\alpha_1, \alpha_2, \ldots \ldots \alpha_u;\ \beta_1, \beta_2, \ldots \ldots \ldots, \beta_v; A_1, A_2, \ldots \ldots A_u \ and\ B_1, B_2, \ldots \ldots \ldots B_v \ (u, v \in \mathbb{N})$ such that

$$1 + \sum_{j=1}^{\infty} B_j - \sum_{j=1}^{\infty} A_j \geq 0 \, , \tag{2}$$

The Wright's generalized hypergeometric function [2]

$${}_uW_v[(\alpha_1, A_1), \ldots, (\alpha_u, A_u); (\beta_1, A_1), \ldots, (\beta_v, A_v)] = {}_uW_v\left[\left(\alpha_j, A_j\right)_{1,u}; \left(\beta_j, B_j\right)_{1,v}; z\right]$$

is defined by

$${}_uW_v\left[\left(\alpha_j, A_j\right)_{1,u}; \left(\beta_j, B_j\right)_{1,v}; z\right] = \sum_{k=0}^{\infty} \left\{\frac{\prod_{j=1}^{u} \Gamma(\alpha_j + kA_j)}{\prod_{j=1}^{v} \Gamma(\beta_j + kB_j)}\right\} . \frac{z^k}{k!} \quad (z \in \mathbb{U})$$

In particular if $A_j = 1 (j = 1,2, \ldots, u)$ and $B_j = 1 (j = 1,2, \ldots, v)$, we have the following obvious relationship

$$\sigma \, {}_uW_v\left[\left(\alpha_j, 1\right)_{1,u}; \left(\beta_j, 1\right)_{1,v}; z\right] = {}_uF_v\,(\alpha_1, \alpha_2, \ldots \ldots \alpha_u;\ \beta_1, \beta_2, \ldots \ldots \ldots, \beta_v; z) \tag{3}$$

Where ${}_uF_v\,(\alpha_1, \alpha_2, \ldots \ldots \alpha_u;\ \beta_1, \beta_2, \ldots \ldots \ldots, \beta_v; z)$ is the generalized hypergeometric function for detail [see 5,8] and

$$\sigma = \left\{\prod_{j=1}^{u} \Gamma(\alpha_j)\right\}^{-1} . \left\{\prod_{j=1}^{v} \Gamma(\beta_j)\right\} \tag{4}$$

Recently Wright hypergeometric functions have been involved in the geometric function theory also for more detail [see 7, 8, 1, 6] here we introduce the linear operator

$$\Theta_p\left[\left(\alpha_j, A_j\right)_{1,u}; \left(\beta_j, B_j\right)_{1,v}\right] : A_p \longrightarrow A_p$$

defined by convolution

$$\Theta_p\left[\left(\alpha_j, A_j\right)_{1,u}; \left(\beta_j, B_j\right)_{1,v}\right] f(z) = \sigma \left\{z^p \ {}_uW_v\left[\left(\alpha_j, 1\right)_{1,u}; \left(\beta_j, 1\right)_{1,v}; z\right]\right\} * f(z)$$

In particular, the operator $\Theta_p\left[\left(\alpha_j, A_j\right)_{1,u}; \left(\beta_j, B_j\right)_{1,v}\right]$ was investigated by Dziok and Raina [1]. We observe that for a function f defined by (1) we have

$$\Theta_p\left[\left(\alpha_j, A_j\right)_{1,u}; \left(\beta_j, B_j\right)_{1,v}\right] f(z) = z^p + \sum_{k=p+n}^{\infty} \Omega_k \, \alpha_k z^k \tag{5}$$

Where

$$\Omega_k = \sigma \left\{\frac{\prod_{j=1}^{u} \Gamma(\alpha_j + A_j(k-p))}{\prod_{j=1}^{v} \Gamma(\beta_j + B_j(k-p))}\right\} . \frac{1}{(k-p)!} \tag{6}$$

Where σ is given by (5). For convenience, we write

$$\Theta_p f(z) = \Theta_p\left[\left(\alpha_j, A_j\right)_{1,u}; \left(\beta_j, B_j\right)_{1,v}\right] f(z) \tag{7}$$

Guney *et al*. [9] defined the general differential operator $D_{\lambda,l,p,\delta,\beta}^{m,\gamma}$ as follows:

$$D^0 f(z) = f(z)$$

$$D^{1,\gamma}_{\lambda,l,p,\delta,\beta} f(z) = \frac{p-p(\lambda-\delta)(\beta-\gamma)+l}{p+l}\, \Theta_p f(z) + \frac{(\lambda-\delta)(\beta-\gamma)}{p+l}\, z\left(\Theta_p f(z)\right)' = D^{\gamma}_{\lambda,l,p,\delta,\beta} f(z) \tag{8}$$

$$D^{2,\gamma}_{\lambda,l,p,\delta,\beta} f(z) = D^{\gamma}_{\lambda,l,p,\delta,\beta}\left(D^{1,\gamma}_{\lambda,l,p,\delta,\beta} f(z)\right)$$

.

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$$D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f(z) = D^{\gamma}_{\lambda,l,p,\delta,\beta}\left(D^{m-1,\gamma}_{\lambda,l,p,\delta,\beta} f(z)\right) \tag{9}$$

Where $\lambda, l, p, \delta, \beta \geq 0, \lambda > \delta, \beta > \gamma,\ m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

If f(z) is given by (1), then by (5) - (9), we get

$$D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f(z) = z^p + \sum_{k=p+n}^{\infty} \Omega_k^m\, \alpha_k z^k, \tag{10}$$

Where

$$\Omega_k^m = \sigma \left\{\frac{\prod_{j=1}^{u} \Gamma(\alpha_j + A_j(k-p))}{\prod_{j=1}^{v} \Gamma(\beta_j + B_j(k-p))}\right\} \cdot \frac{1}{(k-p)!} \cdot \frac{p+(k-p)(\lambda-\delta)(\beta-\gamma)+l}{p+l} \tag{11}$$

For the special cases of some variables of the operator $D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f(z)$, we have $D^{m,\gamma}_{\lambda,l,p} f(z)$ which was introduced and studied by [10] (see for details [9])

In [9], the author defined the class $S^{m,\gamma,p}_{\lambda,l,\delta,\beta}(n,\varphi,b)$ be the class of function $f(z) \in A_p$.

$$\Re\left\{p + \frac{1}{b}\left(\frac{z\left(D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f(z)\right)'}{D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f(z)} - p\right)\right\} > \varphi \tag{12}$$

And $C^{m,\gamma,p}_{\lambda,l,\delta,\beta}(n,\varphi,b)$ be the class of function $f(z) \in A_p$ satisfying

$$\Re\left\{p + \frac{1}{b}\left(1 + \frac{z\left(D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f(z)\right)''}{\left(D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f(z)\right)'} - p\right)\right\} > \varphi \tag{13}$$

Where $z \in \mathbb{U},\ b \in \mathbb{C} - \{0\}, 0 \leq \varphi < p$ and $D^{m,\gamma}_{\lambda,l,p,\delta,\beta}$ is the differential operator defined by (10).

We note that $f \in C^{m,\gamma,p}_{\lambda,l,\delta,\beta}(n,\varphi,b)$ if and only if $\frac{1}{p}\, z f' \in S^{m,\gamma,P}_{\lambda,l,\delta,\beta}(n,\varphi,b)$.

We define the class $M^{m,\gamma,p}_{\lambda,l,\delta,\beta}(n,\varphi,\alpha,b,s)$ of the function $f(z) \in A_p$ which satisfy the following inequality

$$\Re\left\{p + \frac{1}{b}\left(\frac{z\left(\frac{1-\alpha}{1+s(p+\alpha-1)}\left[(1-s)D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f(z) + sz\left(D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f(z)\right)'\right]\right)'}{\frac{1-\alpha}{1+s(p+\alpha-1)}\left[(1-s)D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f(z) + sz\left(D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f(z)\right)'\right]} - p\right)\right\} > \varphi \tag{14}$$

Where $z \in \mathbb{U},\ b \in \mathbb{C} - \{0\}, 0 \leq \varphi < p$ and $D^{m,\gamma}_{\lambda,l,p,\delta,\beta}$ is the differential operator defined by (10).

Now we define general integral operator as follows:

$$D^{q,\gamma,\alpha}_{\lambda,l,p,\delta,\beta}\, \mathcal{F}^{m,\gamma,p,s}_{\lambda,l,p,\delta,\beta,r,[|\mu|]}(z) = \int_0^z p\, t^{p-1} \prod_{i=1}^{[|\mu|]} \left(\frac{(1-\alpha)(1-s_i)}{1+s_i(p+\alpha-1)} \frac{D^{m_i\,,\,\gamma}_{\lambda,l,p,\delta,\beta} f_i(t)}{t^p} + \frac{(1-\alpha)s_i}{1+s_i(p+\alpha-1)} \frac{\left(D^{m_i\,,\,\gamma}_{\lambda,l,p,\delta,\beta} f_i(t)\right)'}{t^{p-1}}\right)^{r_i} dt, \tag{15}$$

Where $z \in \mathbb{U}, f_i \in A_p, \mu \in \mathbb{C},\ |\mu| \notin [0,1), [|\mu|]$ is the integral part of the modulus of μ, $I = [0,1]$, $s = (s_1, s_2, \dots, s_{[|\mu|]}) \in I^{[|\mu|]}, m = (m_1, m_2, \dots, m_{[|\mu|]}) \in \mathbb{N}_0^{[|\mu|]},\ r = (r_1, r_2, \dots, r_{[|\mu|]}) \in \mathbb{R}_+^{[|\mu|]}$ and $D^{q,\gamma}_{\lambda,l,p,\delta,\beta}$ is the differential operator defined by (10).

Remark 1: Taking $s_1 = s_2 = \dots = s_{[|\mu|]} = 0$ and $s_1 = s_2 = \dots = s_{[|\mu|]} = 1$ with the special cases of some variables of the operator $D^{q,\gamma}_{\lambda,l,p,\delta,\beta}$, we obtain the general integral operator $D^{q,\alpha}_{\lambda,l,p}\mathcal{F}_{p,[|\mu|],m,k}$ and $D^{q,\alpha}_{\lambda,l,p}\mathcal{G}_{p,[|\mu|],m,k}$ which were studied in [11], respectively.

Remark2: Taking $s_1 = s_2 = \dots = s_{[|\mu|]} = 0$ and $s_1 = s_2 = \dots = s_{[|\mu|]} = 1, \alpha = 0, q = 0, \mu = c \in \mathbb{N}, \alpha_j = A_j = 1$ $(j = 1,2,\dots,u),\ \beta_j = B_j = 1\ (j = 1,2,\dots,v), u - v = 1, \lambda = 1, \beta = 1, \delta = 0, \gamma = 0, l = 0$ we obtain

$$\mathcal{F}_{p,c,m,k}(z) = \int_0^z p t^{p-1} \left(\frac{D^{m_1} f_1(t)}{t^p}\right)^{k_1} \dots\dots \left(\frac{D^{m_c} f_c(t)}{t^p}\right)^{k_c} dt$$

And

$$\mathcal{G}_{p,c,m,k}(z) = \int_0^z p t^{p-1} \left(\frac{D^{m_1} f_1(t)}{p t^{p-1}}\right)^{k_1} \dots\dots \left(\frac{D^{m_c} f_c(t)}{p t^{p-1}}\right)^{k_c} dt$$

Which was introduced by Saltik *et al*. [12], respecticely.

Remark3: Taking $s_1 = s_2 = \dots = s_{[|\mu|]} = 0$ and $s_1 = s_2 = \dots = s_{[|\mu|]} = 1, \alpha = 0, q = 0,\ \mu = c \in \mathbb{N}, \alpha_j = A_j = 1\ (j = 1,2,\dots,u),\ \beta_j = B_j = 1\ (j = 1,2,\dots,v), u - v = 1,\ \beta = 1, \delta = 0, \gamma = 0, l = 0,\ m_1 = m_2 = \dots = m_{[|\mu|]} = 0,\ \lambda = 1$, we obtain

$$\mathcal{F}_p(z) = \int_0^z p t^{p-1} \left(\frac{f_1(t)}{t^p}\right)^{k_1} \dots\dots \left(\frac{f_c(t)}{t^p}\right)^{k_c} dt$$

And

$$\mathcal{G}_p(z) = \int_0^z pt^{p-1} \left(\frac{f_1(t)}{pt^{p-1}}\right)^{k_1} \dots\dots \left(\frac{f_c(t)}{pt^{p-1}}\right)^{k_c} dt$$

Which was introduced by Frasin [13], respecticely.

2. Sufficient conditions of the operator $D^{q,\gamma,\alpha}_{\lambda,l,p,\delta,\beta}\, \mathcal{F}^{m,\gamma,p,s}_{\lambda,l,p,\delta,\beta,r,[|\mu|]}$

For the simplicity, we will use the symbol $D\mathcal{F}^q$ instead of $D^{q,\gamma,\alpha}_{\lambda,l,p,\delta,\beta}\, \mathcal{F}^{m,\gamma,p,s}_{\lambda,l,p,\delta,\beta,r,[|\mu|]}$.

Theorem 1: Let $\mu \in \mathbb{C}$, $|\mu| \notin [0,1)$, $[|\mu|]$ is the integral part of the modulus of μ, $s = (s_1, s_2, \dots, s_{[|\mu|]}) \in I^{[|\mu|]}$, $I = [0,1]$, $m = (m_1, m_2, \dots, m_{[|\mu|]}) \in \mathbb{N}_0^{[|\mu|]}$, $r = (r_1, r_2, \dots\dots, r_{[|\mu|]}) \in \mathbb{R}_+^{[|\mu|]}$. Also $f_i \in M^{m,\gamma,p}_{\lambda,l,\delta,\beta}(n, \varphi_i, \alpha, b, s)$ for $1 \le i \le [|\mu|]$. if

$$0 \le p + \sum_{i=1}^{[|\mu|]} r_i(\varphi_i - p) < p, \qquad (16)$$

Then the integral operator $D\mathcal{F}^q$ defined by (15) is in the class $M^{m,\gamma,p}_{\lambda,l,\delta,\beta}(n, \xi, \alpha, b, s)$, where

$$\xi = p + \sum_{i=1}^{[|\mu|]} r_i(\varphi_i - p). \qquad (17)$$

Proof: From (15), we obtain

$$(D\mathcal{F}^q)'(z) = p\, z^{p-1} \prod_{i=1}^{[|\mu|]} \left(\frac{(1-\alpha)(1-s_i)}{1+s_i(p+\alpha-1)} \frac{D^{m_i,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z)}{z^p} + \frac{(1-\alpha)s_i}{1+s_i(p+\alpha-1)} \frac{\left(D^{m_i,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z)\right)'}{z^{p-1}} \right)^{r_i}$$

and

$$\frac{z(D\mathcal{F}^q)''(z)}{(D\mathcal{F}^q)'(z)} = (p-1) + \sum_{i=1}^{[|\mu|]} r_i \left(\frac{z\left(\frac{1-\alpha}{1+s_i(p+\alpha-1)}\left[(1-s_i)D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z) + s_i z \left(D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z)\right)'\right]\right)'}{\frac{1-\alpha}{1+s_i(p+\alpha-1)}\left[(1-s_i)D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z) + s_i z \left(D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z)\right)'\right]} - p \right)$$

Or equivalently

$$\frac{z(D\mathcal{F}^q)''(z)}{(D\mathcal{F}^q)'(z)} + 1 - p = \sum_{i=1}^{[|\mu|]} r_i \left(\frac{z\left(\frac{1-\alpha}{1+s_i(p+\alpha-1)}\left[(1-s_i)D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z) + s_i z \left(D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z)\right)'\right]\right)'}{\frac{1-\alpha}{1+s_i(p+\alpha-1)}\left[(1-s_i)D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z) + s_i z \left(D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z)\right)'\right]} - p \right)$$

That is

$$\frac{1}{b}\left(\frac{z(D\mathcal{F}^q)''(z)}{(D\mathcal{F}^q)'(z)} + 1 - p\right) = \sum_{i=1}^{[|\mu|]} r_i \frac{1}{b} \left(\frac{z\left(\frac{1-\alpha}{1+s_i(p+\alpha-1)}\left[(1-s_i)D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z) + s_i z \left(D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z)\right)'\right]\right)'}{\frac{1-\alpha}{1+s_i(p+\alpha-1)}\left[(1-s_i)D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z) + s_i z \left(D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z)\right)'\right]} - p \right)$$

Or equivalently

$$\frac{1}{b}\left(\frac{z(D\mathcal{F}^q)''(z)}{(D\mathcal{F}^q)'(z)} + 1 - p\right) + p$$
$$= \sum_{i=1}^{[|\mu|]} r_i \left(p + \frac{1}{b} \left(\frac{z\left(\frac{1-\alpha}{1+s_i(p+\alpha-1)}\left[(1-s_i)D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z) + s_i z \left(D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z)\right)'\right]\right)'}{\frac{1-\alpha}{1+s_i(p+\alpha-1)}\left[(1-s_i)D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z) + s_i z \left(D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z)\right)'\right]} - p \right) \right) + p - p\sum_{i=1}^{[|\mu|]} r_i$$

Since $f_i \in M^{m,\gamma,p}_{\lambda,l,\delta,\beta}(n, \varphi_i, \alpha, b, s)$ $(1 \le i \le [|\mu|]$, we get

$$\Re\left(\frac{1}{b}\left(\frac{z(D\mathcal{F}^q)''(z)}{(D\mathcal{F}^q)'(z)} + 1 - p\right) + p\right)$$
$$= \sum_{i=1}^{[|\mu|]} r_i \Re\left(p + \frac{1}{b} \left(\frac{z\left(\frac{1-\alpha}{1+s_i(p+\alpha-1)}\left[(1-s_i)D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z) + s_i z \left(D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z)\right)'\right]\right)'}{\frac{1-\alpha}{1+s_i(p+\alpha-1)}\left[(1-s_i)D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z) + s_i z \left(D^{m,\gamma}_{\lambda,l,p,\delta,\beta} f_i(z)\right)'\right]} - p \right) \right) + p - p\sum_{i=1}^{[|\mu|]} r_i$$
$$> p + \sum_{i=1}^{[|\mu|]} r_i(\varphi_i - p)$$

Hence the integral operator $D^{q,\gamma,\alpha}_{\lambda,l,p,\delta,\beta}\, \mathcal{F}^{m,\gamma,p,s}_{\lambda,l,p,\delta,\beta,r,[|\mu|]}(z) \in M^{m,\gamma,p}_{\lambda,l,\delta,\beta}(n, \xi, \alpha, b, s)$ with

$$\xi = p + \sum_{i=1}^{[|\mu|]} r_i(\varphi_i - p)$$

Which completes proof of theorem.

Corollary 1: Let $\mu \in \mathbb{C}$, $|\mu| \notin [0,1)$, $[|\mu|]$ is the integral part of the modulus of μ, $s_1 = s_2 = \dots = s_{[|\mu|]} = 0$, $m = (m_1, m_2, \dots, m_{[|\mu|]}) \in \mathbb{N}_0^{[|\mu|]}$, $r = (r_1, r_2, \dots\dots, r_{[|\mu|]}) \in \mathbb{R}_+^{[|\mu|]}$. Also $f_i \in S^{m,\gamma,p}_{\lambda,l,\delta,\beta}(n, \varphi_i, b)$ for $1 \le i \le [|\mu|]$. if

$$0 \le p + \sum_{i=1}^{[|\mu|]} r_i(\varphi_i - p) < p,$$

Then the integral operator

$$D^{q,\gamma,\alpha}_{\lambda,l,p,\delta,\beta}\,\mathcal{F}^{m,\gamma,p,s}_{\lambda,l,p,\delta,\beta,r,[|\mu|]}(z) = \int_0^z p\,t^{p-1}\prod_{i=1}^{[|\mu|]}\left((1-\alpha)\frac{D^{m_i\,,\,\gamma}_{\lambda,l,p,\delta,\beta}\,f_i(t)}{t^p}\right)^{r_i} dt$$

is in the class $S^{m,\gamma,p}_{\lambda,l,\delta,\beta}(n,\xi,b)$, where $\xi = p + \sum_{i=1}^{[|\mu|]} r_i(\varphi_i - p)$.

Corollary 2: Let $\mu \in \mathbb{C}$, $|\mu| \notin [0,1)$, $[|\mu|]$ is the integral part of the modulus of μ, $s_1 = s_2 = \ldots = s_{[|\mu|]} = 1$, $m = (m_1, m_2, \ldots, m_{[|\mu|]}) \in \mathbb{N}_0^{[|\mu|]}$, $r = (r_1, r_2, \ldots, r_{[|\mu|]}) \in \mathbb{R}_+^{[|\mu|]}$. Also $f_i \in S^{m,\gamma,p}_{\lambda,l,\delta,\beta}(n,\varphi_i,b)$ for $1 \le i \le [|\mu|]$. if

$$0 \le p + \sum_{i=1}^{[|\mu|]} r_i(\varphi_i - p) < p\,,$$

Then the integral operator

$$D^{q,\gamma,\alpha}_{\lambda,l,p,\delta,\beta}\,\mathcal{G}^{m,\gamma,p,s}_{\lambda,l,p,\delta,\beta,r,[|\mu|]}(z) = \int_0^z p\,t^{p-1}\prod_{i=1}^{[|\mu|]}\left(\frac{(1-\alpha)}{(p+\alpha)}\frac{\left(D^{m_i\,,\,\gamma}_{\lambda,l,p,\delta,\beta}\,f_i(t)\right)'}{t^{p-1}}\right)^{r_i} dt$$

is in the class $C^{m,\gamma,p}_{\lambda,l,\delta,\beta}(n,\xi,b)$, where $\xi = p + \sum_{i=1}^{[|\mu|]} r_i(\varphi_i - p)$.

Remark4: Taking $s_1 = s_2 = \cdots = s_{[|\mu|]} = 0$ and $s_1 = s_2 = \ldots = s_{[|\mu|]} = 1, \alpha = 0, q = 0, \mu = c \in \mathbb{N}, \alpha_j = A_j = 1$ $(j = 1,2,\ldots,u)$, $\beta_j = B_j = 1$ $(j = 1,2,\ldots,v), u - v = 1$, $\beta = 1, \delta = 0, \gamma = 0, l = 0$, $m_1 = m_2 = \ldots = m_{[|\mu|]} = 0$, $\lambda = 1, \alpha = 0$ and $p = n - 1$ in Theorem 1, we have Theorem 1 and Theorem 3 in [3] respectively. Also adding $\varphi_1 = \varphi_2 = \ldots\ldots = \varphi_{[|\mu|]} = \varphi$ to the hypothesis, we obtain Theorem 1 and Theorem 3 in [4] respectively.

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