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Common fixed point theorem of weakly compatible mapping in Intuitionistic Menger spaces with the property E.A

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Abstract
In this paper, we prove some common fixed point theorems for weakly compatible mappings in intuitionistic Menger space using the common property (E.A.) for four finite families of self-mappings. Our results is generalize intuitionistic Menger space are the version of intuitionistic fuzzy metric space.

Keywords: Intuitionistic Menger space, Weak Compatibility, property (E.A.), common property (E.A.) Common fixed point

1. Introduction

The notion of probabilistic metric spaces (briefly, PM-space) as a generalization of metric spaces, was introduced by K, Menger [21] in 1942. The idea of K. Menger was to use distribution function instead of non-negative real numbers as values of the metric. The notion of a probabilistic metric space corresponds to situations when we do not know exactly the distance between two points, but we know probabilities of possible values of this distance. Schweizer and Sklar [31, 32] studied this concept and gave some fundamental results on this space. It is observed by many authors that contraction condition in metric space may be exactly translated in to PM-space endowed with the min norm. In 1972, V.M. obtained a generalization of Banach contraction principle on a complete Menger space which is mile stone in development fixed point theory in Menger space. In 1986 ha-Reid [33] obtained a generalization of Banach contraction principle on a complete Menger space which is mile stone in development fixed point theory in Menger space. In 1986, Jungck [13] introduced the notion of compatible maps for a pair of self-maps in metric space. Jungck and Rohades [14] termed a pair of self-maps to be coincidently commuting or equivalently weak-compatible if they commute at their coincidence points.

Modifying the idea of Kramosil and Michalek [16], George and Veeramani [10] introduced fuzzy metric space. Recently Park [29] introduced the notion of intuitionistic fuzzy metric spaces as a generalization of fuzzy metric spaces. Kutukcu *et al.* [19] introduced the notion of intuitionistic Menger spaces with the help of t-norms and t-conorms as a generalization of Menger space due to Menger [21]. Further they introduced the notion of Cauchy sequences and found a necessary and sufficient condition for an intuitionistic Menger space to be complete.

Jungck and Rhoades [13] weakned the notion of compatibility by introducing the notion of weakly compatible mappings (extended by Singh and Jain [34] to probabilistic metric space) and proved common fixed point theorems without assuming continuity of the involved mapping in metric space. In 2002 Aamri and Moutawakil [1] introduced the notion of property (E.A.) (extended by Kubiaczyk and Sharma [16] to probabilistic metric space) for self-mappings which contained the class of non-compatible mappings due to Pant [28]. Further, Liu *et al.* [20] defined the notion of common property (E.A.) (extended by Ali *et al.* [3] to probabilistic metric space) which contains the property (E.A.) and proved several fixed point theorems under hybrid contractive conditions. Subsequently, there are a number of results which contained the notions of property (E.A) and common property (E.A.) in Menger spaces [4, 7, 8]. Inspired by Sintunavarat and Kumam [35, 36], Wadhwa *et al.* [37] defined the notion of (E.A.) like property and common (E.A.) like property in fuzzy metric spaces and improved the results of Kumar [19] as the conditions on containment of ranges amongst the involved mappings and closedness of the underlying subspaces are completely relaxed. Many mathematician proved various fixed point theorem in Menger spaces [6, 12, 22, 23, 25, 30].

In this present paper, we prove some common fixed point theorems for weakly compatible mappings in intuitionistic Menger space using the common property (E.A.). In this paper we generalize the result of Kumar S [17]. From intuitionistic fuzzy metric space to intuitionistic Menger space.

2. Preliminaries

Definition 2.1. [31] A binary operation $*$: [0, 1] x [0, 1] →[0, 1] is called a t- norm if $*$ is satisfying the following conditions:

1. $*$ is commutative and associative,
2. $*$ is continuous

3. $a * 1 = a$, for all $a \in [0,1]$,
4. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0,1]$.

Definition 2.2. [31] A binary operation $\diamond: [0,1] \times [0,1] \to [0,1]$ is called a t-conorm if $\diamond$ is satisfying the following conditions:

1. $\diamond$ is commutative and associative,
2. $\diamond$ is continuous
3. $a \diamond 1 = a$, for all $a \in [0,1]$,
4. $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0,1]$.

Remark 2.3: The concept of triangular norms (t-norms) and triangular conorrms (t-conorms) are known as the axiomatic skeletons that we use for characterizing fuzzy intersection and union respectively. These concepts were originally introduced by Menger [22] in his study of statistical metric spaces.

Definition 2.4. [32] A distance distribution function is a function $F : R \to R^+$ which is left continuous on R, non-decreasing and $inf_{t\in R} F(t) = 0,\ sup_{t\in R} F(t) = 1$. We will denote by D the family of all distance distribution functions and by H a special of D defined by

$$H(t) = \begin{cases} 0, & if\ t \leq 0, \\ 1, & if\ t > 0. \end{cases}$$

If X is a non-empty set, $F : X \times X \to D$ is called a probabilistic distance on X and $F(x,y)$ usually denoted by $F_{x,y}$.

Definition 2.5. [32] A non-distance distribution function is a function $L : R \to R^+$ which is right continuous on R, non-decreasing and $inf_{t\in R} L(t) = 1,\ sup_{t\in R} L(t) = 0$. We will denote by E the family of all distance distribution functions and by G a special of E defined by

$$G(t) = \begin{cases} 1, & if\ t \leq 0, \\ 0, & if\ t > 0. \end{cases}$$

If X is a non-empty set, $L : X \times X \to E$ is called a probabilistic non-distance on X and $L(x,y)$ usually denoted by $L_{x,y}$.

Definition 2.6. [19] A 5-tuple $(X, F, L, *, \diamond)$ is said to be an intuitionistic Menger space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is continuous t-conorm, F is a probabilistic distance and L is a probabilistic non-distance on X satisfying the following conditions: for all $x, y, z \in X$ and $t, s \geq 0$.

1. $F_{x,y}(t) + L_{x,y}(t) \leq 1$,
2. $F_{x,y}(0) = 0$,
3. $F_{x,y}(t) = H(t)$ if and only if $x = y$,
4. $F_{x,y}(t) = F_{y,x}(t)$,
5. $if\ F_{x,y}(t) = 1$ and $F_{y,z}(s) = 1$, then $F_{x,z}(t+s) = 1$,
6. $F_{x,z}(t+s) \geq F_{x,y}(t) * F_{y,z}(s)$,
7. $L_{x,y}(0) = 1$,
8. $L_{x,y}(t) = G(t)$ if and only if $x = y$,
9. $L_{x,y}(t) = L_{y,x}(t)$,
10. $if\ L_{x,y}(t) = 0$ and $L_{y,z}(s) = 0$, then $L_{x,z}(t+s) = 0$
11. $L_{x,z}(t+s) \leq L_{x,y}(t) \diamond L_{y,z}(s)$

The function $F_{x,y}(t)$ and $L_{x,y}(t)$ denote the degree of nearness and degree of non-nearness between x and y with respect to t, respectively'

Remark [2.7]: Every Menger space $(X, F, *)$ is intuitionistic Menger space of the form $(X, F, 1-F, *, \diamond)$ such that t-norm $*$ and t-conorm $\diamond$ are associated [11], that is $x \diamond y = 1 - (1-x) * (1-y)$ for any $x, y \in X$.

Example: Let (X, d) be a metric spaces. Then the metric d induces a distance distribution function F defined by

$F_{x,y}(t) = H(t - d(x,y))$ and a non-distance distribution function L defined by $L_{x,y}(t) = G(t - d(x,y))$ for all $x, y \in X$ and $t \geq 0$. Then (X, F, L) is an intuitionistic probabilistic metric space. We call this intuitionistic probabilistic metric space induced by a metric d the induced intuitionistic probabilistic metric space. If t-norm $*$ is $a * b = max.\{a + b - 1, 0\}$ and t-conorm $\diamond$ is $a \diamond b = \min.\{1, a + b\}$ for all $a, b \in [0,1]$ then $(X, F, L, *, \diamond)$ is an intuitionistic Menger space.

Definition 2.8 [19]: Let $(X, F, L, *, \diamond)$ be an intuitionistic Menger space with $t * t \geq t$ and $(1-t) \diamond (1-t) \leq (1-t)$. Then:

(1) A sequence $\{x_n\}_n$ in X is said to be convergent to x in X if, for every $\varepsilon > 0$ and $\lambda \in (0,1)$, there exists positive integer N such that $F_{x_n,x}(\varepsilon) > 1 - \lambda$ and $L_{x_n,x}(\varepsilon) < \lambda$ whenever $n \geq N$.

(2) A sequence $\{x_n\}_n$ in X is called Cauchy sequence if, for every $\varepsilon > 0$ and $\lambda \in (0,1)$, there exists positive integer N such that $F_{x_n,x_m}(\varepsilon) > 1 - \lambda$ and $L_{x_n,x_m}(\varepsilon) < \lambda$ whenever $n, m \geq N$.

(3) an intuitionistic Menger space $(X, F, L, *, \diamond)$ is said to be complete if and only if every Cauchy sequence in X is convergent to a point in X.

Lemma 2.9 [26]: Let $(X, F, L, *, \diamond)$ be an intuitionistic Menger space with $t * t \geq t$ and $(1-t) \diamond (1-t) \leq (1-t)$ and for all $x, y \in X$ and if for a number $k \in (0,1)$ and

$$F_{x,y}(kt) \geq F_{x,y}(t) \text{ and } L_{x,y}(kt) \leq L_{x,y}(t) \quad (1)$$

Then $x = y$.

Proof: Since $t > 0$ and $k \in (0,1)$, we get $t > kt$. In intuitionistic Menger space $(X, F, L, *, \diamond)$, $F_{x,y}$ is non-decreasing and $L_{x,y}$ is non-increasing for all $x, y \in X$, we have
$F_{x,y}(t) \geq F_{x,y}(kt)$ and $L_{x,y}(t) \geq L_{x,y}(kt)$
Using (1) and the definition of intuitionistic Menger space, we have $x = y$.

Definition 2.10 [26]: Two self-maps A and B of an intuitionistic Menger space $(X, F, L, *, \diamond)$ are said to be compatible if for all $t > 0$,
$\lim_{n\to\infty} F_{ABx_n, BAx_n}(t) = 1$ and $\lim_{n\to\infty} L_{ABx_n, BAx_n}(t) = 0$,
Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some $z \in X$.

Definition 2.11[26]: Two self maps A and B of an intuitionistic Menger space $(X, F, L, *, \diamond)$ are said to be weakly compatible if they commute at their coincidence points, that is $Ax = Bx$ for some $z \in X$ then $ABx = BAx$.
Remark: If self maps A and B of an intuitionistic Menger space $(X, F, L, *, \diamond)$ are compatible then they are weakly compatible.

Definition 2.12 [16]: A pair (A, S) of self-mappings of a intuitionistic Menger space $(X, F, L, *, \diamond)$ is said to satisfy the property $(E.A.)$ if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z \in X$$

Clearly, a pair of compatible mappings as well as non compatible mappings satisfies the property E.A.

Example: Let $X = [0,2]$ with the usual metric d, that is, $d(x,y) = |x - y|$ and for each $t \in [0,1]$ define

$$F_{x,y}(t) = \begin{cases} \frac{|x-y|}{t+|x-y|} & if\ t > 0 \\ 0 & if\ t = 0 \end{cases}$$

$$L_{x,y}(t) = \begin{cases} \frac{t}{t+|x-y|} & if\ t > 0 \\ 1 & if\ t = 0 \end{cases}$$

For all $x, y \in X$. then $(X, F, L, *, \diamond)$ is intuitionistic Menger space.

Define self-maps A and S as follows:

$$A(X) = \begin{cases} 1 + x & if\ x \in [0,1] \\ \frac{x}{2} & if\ x \in (1,2] \end{cases}$$

$$S(X) = \begin{cases} 1 - x & if\ x \in [0,1] \\ x & if\ x \in (1,2] \end{cases}$$

consider a sequence $\{x_n\} = \frac{1}{n}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} \left\{1 + \frac{1}{n}\right\} = 1 \in A(X)$$

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} \left\{1 - \frac{1}{n}\right\} = 1 \in S(X)$$

Hence the pair (A, S) satisfies the (E.A.) like property.

Definition 2.13 : Two pairs (A, S) and (B, T) of self-mappings of a intuitionistic Menger space $(X, F, L, *, \diamond)$ are said to satisfy the common property (E.A.) if there exists two sequences $\{x_n\}, \{y_n\}$ in X and some z in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z$$

Example: Let $X = [-2, 2]$ with the usual metric d, that is, $d(x, y) = |x - y|$ and for each $t \in [0,1]$ define

$$F_{x,y}(t) = \begin{cases} \frac{|x-y|}{t+|x-y|} & if\ t > 0 \\ 0 & if\ t = 0 \end{cases}$$

$$L_{x,y}(t) = \begin{cases} \frac{t}{t+|x-y|} & if\ t > 0 \\ 1 & if\ t = 0 \end{cases}$$

For all $x, y \in X$. Then $(X, F, L, *, \diamond)$ is intuitionistic Menger space.

Define self-maps A, B, S and T as follows:

$A(X) = -x$, $B(X) = x$, $(X) = -\frac{x}{2}$, $T(X) = \frac{x}{2}$ for all $x \in X$.

Taking the sequence $\{x_n\} = -\frac{1}{n}$ and $\{y_n\} = \frac{1}{n}$ in X.

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = 0$$

Therefore (A, S) and (B, T) satisfies the common property (E.A.).

Results

In [17] Kumar proved the following result

Theorem [3.2] Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. Further let (A, S) and (B, T) be weakly compatible pairs of self maps of X satisfying

(1) $A(X) \subseteq T(X)$ or $B(X) \subseteq S(X)$
(2) (A, S) or (B, T) satisfies the property $(E.A)$,
(3) There exists $k \in (0,1)$ such that

$$M(Ax, By, kt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t) * M(By, Sx, t)$$

And

$$N(Ax, By, kt) \leq N(Sx, Ty, t) \diamond N(Ax, Sx, t) \diamond N(By, Ty, t) \diamond N(Ax, Ty, t) \diamond N(By, Sx, t)$$

For all $x, y \in X$, and $t > 0$.

If the range of one of the maps A, B, S or T is a complete subspace of X. Then A, B, S and T have a unique common fixed point in X. N

Now we prove the following result

Lemma [3.3] :Let A, B, S and T be self-mapping of a intuitionistic Menger space $(X, F, L, *, \diamond)$ with $*$ is a continuous t-norm and $\diamond$ is continuous t-conorm defined by $t * t \geq t$ and $(1-t) \diamond (1-t) \leq (1-t)$ satisfying the following:

(3.3.1) the pair $(A.S)$ or (B, T) satisfies the property $(E.A.)$

(3.3.2) There exists $k \in (0,1)$ such that for every $x, y \in X$ and $t > 0$

$$F_{Ax,By}(kt) \geq F_{Sx,By}(t) * F_{Ax,Ty}(t) * F_{Sx,Ty}(t) * F_{Ty,By}(t) * F_{Sx,By}(t) * F_{Ty,Ax}(t)$$

And

$$L_{Ax,By}(kt) \leq L_{Sx,By}(t) \diamond L_{Ax,Ty}(t) \diamond L_{Sx,Ty}(t) \diamond$$
$$L_{Ty,By}(t) \diamond L_{Sx,By}(t) \diamond L_{Ty,Ax}(t)$$

(3.3.3) $A(X) \subseteq T(X)$ or $B(X) \subseteq S(X)$

Then the pairs (A, S) and (B, T) share the common property.

Proof: Suppose the pair (A, S) satisfies $E.A.$ property, then there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$.

Since $A(X) \subseteq T(X)$, hence for each $\{x_n\}$ there exists $\{y_n\}$ in X such that $Ax_n = Ty_n$. Therefore $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = z$ Now we claim that $\lim_{n\to\infty} By_n = z$. For providing this applying inequality on $x = x_n, y = y_n$, we obtain

$$F_{Ax_n,By_n}(kt) \geq F_{Sx_n,By_n}(t) * F_{Ax_n,Ty_n}(t) * F_{Sx_n,Ty_n}(t) * F_{Ty_n,By_n}(t) * F_{Sx_n,By_n}(t) * F_{Ty_n,Ax_n}(t)$$
$$and \quad L_{Ax_n,By_n}(kt) \leq L_{Sx_n,By_n}(t) \diamond L_{Ax_n,Ty_n}(t) \diamond L_{Sx_n,Ty_n}(t) \diamond L_{Ty_n,By_n}(t) \diamond L_{Sx_n,By_n}(t) \diamond L_{Ty_n,Ax_n}(t)$$

Taking the limit as $n \to \infty$, we get

$$F_{z,By_n}(kt) \geq F_{z,By_n}(t) * F_{z,z}(t) * F_{z,z}(t) * F_{z,By_n}(t) * F_{z,By_n}(t) * F_{z,z}(t)$$
$$F_{z,By_n}(kt) \geq F_{z,By_n}(t) * 1 * 1 * F_{z,By_n}(t) * F_{z,By_n}(t) * 1$$
$$F_{z,By_n}(kt) \geq F_{z,By_n}(t)$$
$$and \quad L_{z,By_n}(kt) \leq L_{z,By_n}(t) \diamond L_{z,z}(t) \diamond L_{z,z}(t) \diamond L_{z,By_n}(t) \diamond L_{z,By_n}(t) \diamond L_{z,z}(t)$$
$$L_{z,By_n}(kt) \leq L_{z,By_n}(t) \diamond 0 \diamond 0 \diamond L_{z,By_n}(t) \diamond L_{z,By_n}(t) \diamond 0$$
$$L_{z,By_n}(kt) \leq L_{z,By_n}(t)$$

By lemma 2.9 we have

$By_n = z$ and therefore $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z$ for some $z \in X$. Hence the pair (A, S) and (B, T) share the common $E.A.$ property.

Theorem [3.3]: Let A, B, S and T be self-mapping of a intuitionistic Menger space $(X, F, L, *, \diamond)$ with $*$ is a continuous t-norm and $\diamond$ is continuous t-conorm defined by $t * t \geq t$ and $(1-t) \diamond (1-t) \leq (1-t)$ satisfying the condition 3.3.2 and

(3.3.4) The pairs (A, S) and (B, T) share the common $E.A.$ property

(3.3.5) $S(X)$ and $T(X)$ are closed subsets of X.

Then the pairs (A, S) and (B, T) have a point of coincidence each. Moreover A, B, S and T have a unique common fixed point provided both the pairs (A, S) and (B, T) are weakly compatible.

Proof: In view of (3.3.4), there exists two sequence $\{x_n\}$ and $\{y_n\}$ in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z$ for some $z \in X$.

As $S(X)$ is a closed subset of X. Therefore there exists a point $u \in X$ such that $z = Su$. We claim that $Au = z$ put $x = u$ and $y = y_n$ in inequality 3.3.2 we get

$$F_{Au,By_n}(kt) \geq F_{Su,By_n}(t) * F_{Au,Ty_n}(t) * F_{Su,Ty_n}(t) * F_{Ty_n,By_n}(t) * F_{Su,By_n}(t) * F_{Ty_n,Au}(t)$$
$$L_{Au,By_n}(kt) \leq L_{Su,By_n}(t) \diamond L_{Au,Ty_n}(t) \diamond L_{Su,Ty_n}(t) \diamond L_{Ty_n,By_n}(t) \diamond L_{Su,By_n}(t) \diamond L_{Ty_n,Au}(t)$$

Taking the limit as $n \to \infty$, we get

$$F_{Au,z}(kt) \geq F_{z,z}(t) * F_{Au,z}(t) * F_{z,z}(t) * F_{z,z}(t) * F_{z,z}(t) * F_{z,Au}(t)$$
$$F_{Au,z}(kt) \geq 1 * F_{Au,z}(t) * 1 * 1 * 1 * F_{z,Au}(t)$$
$$F_{Au,z}(kt) \geq F_{Au,z}(t)$$

And

$$L_{Au,z}(kt) \leq L_{z,z}(t) \diamond L_{Au,z}(t) \diamond L_{z,z}(t) \diamond L_{z,z}(t) \diamond L_{z,z}(t) \diamond L_{z,Au}(t)$$
$$L_{Au,z}(kt) \leq 0 \diamond L_{Au,z}(t) \diamond 0 \diamond 0 \diamond 0 \diamond L_{z,Au}(t)$$
$$L_{Au,z}(kt) \leq L_{Au,z}(t)$$

By lemma 2.9 we have $Au = z$. Therefore $Au = z = Su$ which shows that u is a coincidence point of the pair (A, S).
Since $T(X)$ is also a closed subset of X, therefore there exists a point $v \in X$ such that $Tv = z$. Now we show that $Bv = z$ put $x = u$ and $y = v$ in inequality 3.3.2 we get

$$F_{Au,Bv}(kt) \geq F_{Su,Bv}(t) * F_{Au,Tv}(t) * F_{Su,Tv}(t) * F_{Tv,Bv}(t) * F_{Su,Bv}(t) * F_{Tv,Au}(t)$$
$$F_{z,Bv}(kt) \geq F_{z,Bv}(t) * F_{z,z}(t) * F_{z,z}(t) * F_{z,Bv}(t) * F_{z,Bv}(t) * F_{z,z}(t)$$
$$F_{z,Bv}(kt) \geq F_{z,Bv}(t) * 1 * 1 * F_{z,Bv}(t) * F_{z,Bv}(t) * 1$$
$$F_{z,Bv}(kt) \geq F_{z,Bv}(t)$$

and

$$L_{Au,Bv}(kt) \leq L_{Su,Bv}(t) \diamond L_{Au,Tv}(t) \diamond L_{Su,Tv}(t) \diamond L_{Tv,Bv}(t) \diamond L_{Su,Bv}(t) \diamond L_{Tv,Au}(t)$$
$$L_{z,Bv}(kt) \leq L_{z,Bv}(t) \diamond L_{z,z}(t) \diamond L_{z,z}(t) \diamond L_{z,Bv}(t) \diamond L_{z,Bv}(t) \diamond L_{z,z}(t)$$
$$L_{z,Bv}(kt) \leq L_{z,Bv}(t) \diamond 0 \diamond 0 \diamond L_{z,Bv}(t) \diamond L_{z,Bv}(t) \diamond 0$$
$$L_{z,Bv}(kt) \leq L_{z,Bv}(t)$$

By lemma 2.9 we have $Bv = z$.Therefore $Tv = z = Bv$ which shows that v is a coincidence point of the pair (B, T). Since the pairs (A, S) and (B, T) are weakly compatible and $Au = Su, Bv = Tv$ therefore $Az = ASu = SAu = Sz$ put $x = z$ and $y = v$ in inequality 3.3.2 we get

$$F_{Az,Bv}(kt) \geq F_{Sz,Bv}(t) * F_{Az,Tv}(t) * F_{Sz,Tv}(t) * F_{Tv,Bv}(t) * F_{Sz,Bv}(t) * F_{Tv,Az}(t)$$
$$F_{Az,z}(kt) \geq F_{Az,z}(t) * F_{Az,z}(t) * F_{Az,z}(t) * F_{z,z}(t) * F_{Az,z}(t) * F_{z,Az}(t)$$
$$F_{Az,z}(kt) \geq F_{Az,z}(t) * F_{Az,z}(t) * F_{Az,z}(t) * 1 * F_{Az,z}(t) * F_{z,Az}(t)$$
$$F_{Az,z}(kt) \geq F_{Az,z}(t)$$

and

$$L_{Az,Bv}(kt) \leq L_{Sz,Bv}(t) \diamond L_{Az,Tv}(t) \diamond L_{Sz,Tv}(t) \diamond L_{Tv,Bv}(t) \diamond L_{Sz,Bv}(t) \diamond L_{Tv,Az}(t)$$
$$L_{Az,z}(kt) \leq L_{Az,z}(t) \diamond L_{Az,z}(t) \diamond L_{Az,z}(t) \diamond L_{z,z}(t) \diamond L_{Az,z}(t) \diamond L_{z,Az}(t)$$
$$L_{Az,z}(kt) \leq L_{Az,z}(t) \diamond L_{Az,z}(t) \diamond L_{Az,z}(t) \diamond 0 \diamond L_{Az,z}(t) \diamond L_{z,Az}(t)$$
$$L_{Az,z}(kt) \leq L_{Az,z}(t)$$

By lemma 2.9 we have $Az = z$, therefore $Az = Sz = z$. Similarly we can prove that $Bz = Tz = z$.Hence $Az = Bz = Sz = Tz = z$ then z is a common fixed point of A, B, S and T.

Uniqueness: Let $w(w \neq z)$ be another common fixed point of A, B, S and T, then $w = Aw = Bw = Sw = Tw$ put $x = z$ and $y = w$ in inequality 3.3.2 we get

$$F_{Az,Bw}(kt) \geq F_{Sz,Bw}(t) * F_{Az,Tw}(t) * F_{Sz,Tw}(t) * F_{Tw,Bw}(t) * F_{Sz,Bw}(t) * F_{Tw,Az}(t)$$
$$F_{z,w}(kt) \geq F_{z,w}(t) * F_{z,w}(t) * F_{z,w}(t) * F_{w,w}(t) * F_{z,w}(t) * F_{w,z}(t)$$
$$F_{z,w}(kt) \geq F_{z,w}(t)$$
$$L_{Az,Bw}(kt) \leq L_{Sz,Bw}(t) \diamond L_{Az,Tw}(t) \diamond L_{Sz,Tw}(t) \diamond L_{Tw,Bw}(t) \diamond L_{Sz,Bw}(t) \diamond L_{Tw,Az}(t)$$
$$L_{z,w}(kt) \leq L_{z,w}(t) \diamond L_{z,w}(t) \diamond L_{z,w}(t) \diamond L_{w,w}(t) \diamond L_{z,w}(t) \diamond L_{w,z}(t)$$
$$L_{z,w}(kt) \leq L_{z,w}(t)$$

By lemma 2.9 we have $z = w$.
Thus z is a unique common fixed point of A, B, S and T.

Corollary 3.5: Let A and S be self-mapping of a intuitionistic Menger space $(X, F, L, *, \diamond)$ with $*$ is a continuous t-norm and $\diamond$ is continuous t-conorm defined by $t * t \geq t$ and $(1-t) \diamond (1-t) \leq (1-t)$ satisfying the following conditions:

(i) The pair (A, S) share the common $E.A.$ property

(ii) There exists $k \in (0,1)$ such that for every $x, y \in X$ and $t > 0$

$$F_{Ax,Ay}(kt) \geq F_{Sx,Ay}(t) * F_{Ax,Sy}(t) * F_{Sx,Sy}(t) * F_{Sy,Ay}(t) * F_{Sx,Ay}(t) * F_{Sy,Ax}(t)$$
$$L_{Ax,Ay}(kt) \leq L_{Sx,Ay}(t) \diamond L_{Ax,Sy}(t) \diamond L_{Sx,Sy}(t) \diamond L_{Sy,Ay}(t) \diamond L_{Sx,Ay}(t) \diamond L_{Sy,Ax}(t)$$

(iii) $S(X)$ is a closed subsets of X.

Then the pairs (A, S) has a point of coincidence each. Moreover A and S have a unique common fixed point provided both the pairs (A, S) are weakly compatible.

References

1. Aamri Moutawakil M. DEL: Some new common fixed point theorems under strict contractive conditions J. Math. Anal. Appl. 2002; 270(1):181-188.
2. Alaca C, Turkoglu D, Yildiz C. fixed points in intuitionistic fuzzy metric space Chaos Solitons & Fractals. 2006; 29:1073-1078.
3. Ali J, Imdad M, Bahaguna D. Common fixed point theorems in Menger spaces with common property (E.A.) Comput. Math. Appl. 2010; 60(12):3152-3159.
4. Ali J, Imdad M, Mihet D, Tanveer M. Common fixed points of strict contractions in Menger spaces, Acta Math. Hungar. 2011; 132(4):367-386.
5. Chang SS, Cho YJ, Kang SM. Nonlinear Operator Theory in Probabilistic Metric spaces. NovaScience Publishers, Huntington, USA, 2001.
6. Chauhan S, Pant BD. Common fixed point theorem for weakly compatible mappings in Menger space, J. Adv. Res. Pure Math. 2011; 3(2):107-109.
7. Fang J-X, Gao Y. Common fixed point theorems under strict contractive conditions in Menger spaces, Nonlinear Anal. 2009; 70(1):184-193.
8. Fang JX. Common fixed point theorems of compatible and weakly compatible maps in Menger spaces, Nonlinear Anal. 2009; 71(5-6):1833-1843.
9. Gopal D, Imdad M. Some new common fixed point theorems in fuzzy metric spaces, Ann. Univ. Ferrara Sez VII Sci. Mat. 2011; 57(2):303-316.
10. George A. and Veeramani P. On some results in fuzzy metric spaces. Fuzzy sets and systems, 1994; 64:395:399.
11. Hadzic O, Pep E. Fixed point theory in probabilistic metric spaces. Dordrecht: Kluwer Acad. Pub. 2001.
12. Imdad M, Ali J, Tanveer M. Remarks on some recent material fixed point theorems, Appl. Math. Lett. 2011; 24(7):1165-1169.
13. Imdad M, Ali J, Tanveer M. Coincidence and common fixed point theorem for nonlinear contractions in Menger PM spaces, Chaos Solitons & Fractals. 2009; 42(5):3121-3129.
14. Jungck G. Compatible mappings and common fixed points.Int. J. math.Math. Sci. 1986; 9:771:773.
15. Jungck G, Rhoades BE. Fixed points for set valued functions without continuity. Indian J. Pure Appl. Math. 1998; 29:227:238.
16. Kramosil O, Michalek J. Fuzzy metric and statistical spaces. Kybernetica, 1975; 11:326:334.
17. Kubiaczyk I, Sharma S. Some common fixed point theorems in Menger space under strict contractive conditions Southeast Asian Bull. Math. 2008; 32(1):117-124. MR2385106 Zbi 119954223.

18. Kumar S. Common fixed point theorems in intuitionistic fuzzy metric space using property (E. A.) J. Indian math. Society. 2009; 76(1-4):94-103.
19. Kumar S. Fixed point theorems for weakly compatible maps under E. A. property in fuzzy metric spaces, J. appl. Math. Inform. 2011; 29(1-2):395-405.
20. Kutukcu S, Tuna A, Yakut AT. Generalized contraction mapping principle in intuitionistic Menger space and application to differential equation, Appl. Math. And Mech. 2007; 28:799-809.
21. Liu Y, Wu j, Li Z. Common fixed points of single –valued and multi-valued maps. Int. J. Math. Math. Sci. 2005; 19:3045-3055.
22. Menger k. Statistical metric. Proc. Nat. Acad. USA. 1942; 28:535:537.
23. Mihet DA. generalization of a contraction principle in Probabilistic metric spaces, Part II, Int. J. Math. Math. Sci. 2005, 729-736.
24. Mihet D. a note on a common fixed point theorem in probabilistic metric spaces, Acta Math. Hungar. 2009; 125(1-2):127-130.
25. Mishra SN, Common fixed points of compatible mappings in PM-spaces, Math. Japon. 1991; 36:283-289.
26. Pant BD, Chauhan S, Alam Q. Common fixed point theorem in probabilistic metric space, Krag. J. math. 2011; 35(3):463-470.
27. Pant BD, Chauhan S, Pant Vaishali. Common fixed point theorem in Intuitionistic Menger Spaces. Journal of Advanced Studies in Topology. 2010; 1:54-62.
28. Pant RP. Common fixed points of four mappings, Bull. Cal. Math. Soc. 1998; 90:281-286.
29. Pant RP. Common fixed point theorems for contractive maps. J. Math. Anal. Appl. 1998; 226:251-258. MR1646430
30. Park JH. Intuitionistic fuzzy metric space, chos, Solutions and Fractals, 2004; 22:1039-1046.
31. Razani A, Shirdaryazdi M. a common fixed point theorem of compatible maps in Menger space Chaos Solitons & Fractals. 2007; 32(1):26-34.
32. Schweizer B, Sklar A. Statistical metric spaces. Pacific J. Math. 1960; 10:313:334.
33. Schweizer B, Sklar A. Probabilistic metric spaces. Elsevier, North-Holland, New York, 1983.
34. Sehgal VM, Bharucha AT. Reid. Fixed point of contraction mappings on probabilistic metric spaces. Math. Syst. Theory, 1972; 6:97:102.
35. Singh B, Jain S. A fixed point theorem in Menger space through weak compatibility. J. Math. Anal. Appl. 2005; 301:439-448.
36. Sintunavarat W, Kumam P. Common fixed point theorems for a pair of weakly compatible mappings in fuzzy metric spaces, J. Appl. Math. 2011, Article ID637958.
37. Sintunavarat W, Kumam P. Common fixed points for R-weakly commuting in fuzzy metric spaces, Ann. Univ. Ferrara Sez. VII Sci Mat. 2012. In press. DOI: 10.1007/s11565-012-0150-z.
38. Wadhwa K, Dubey H, Jain R. Impact of 'E.A. like" property on common fixed point theorems in fuzzy metric spaces, J. Adv. Stud. Topology. 2012; 3(1):52-59.