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On ternary quadratic Diophantine equation $6(x^2+y^2)-11xy+x+y+1=24z^2$

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Abstract

The ternary quadratic equation $6(x^2+y^2)-11xy+x+y+1=24z^2$ analyzed by its nonzero distinct integer point on it. Employing the integer solutions, a few relations between the solutions are presented. Also, knowing an integer solution formula for generating sequence of solutions are given.

Keywords: ternary, quadratic, integer solutions, figurate numbers

1. Introduction

The Ternary quadratic equation offers an unlimited field for research because of their variety [1-2]. In particular, one may refer [3-25] for finding points on some specific three dimensional surfaces. This communication concerns with yet another ternary quadratic equation $6(x^2+y^2)-11xy+x+y+1=24z^2$ for determining its infinitely many integer solutions. Employing integral solutions, a few interesting relations among the solutions are given.

2. Method of Analysis

Consider the equation

$$6(x^2+y^2)-11xy+x+y+1=24z^2 \tag{1}$$

The substitution of linear transformations

$$x=u+v;\ y=u-v \quad (u\neq v\neq 0) \tag{2}$$

in (1) leads to

$$(u+1)^2+23v^2=24z^2 \tag{3}$$

where $u+1=U$

The above equation is solved through different methods and using (2), different patterns of integer solutions to (1) are obtained.

Pattern: 1

Write 24 as

$$24=(1+i\sqrt{23})(1-i\sqrt{23}) \tag{4}$$

Assume

$$z(a,b)=z=a^2+23b^2 \tag{5}$$

where a and b are non - zero integers

Using (4) and (5) in (3) and employing the method of factorization, define

$$(U+i\sqrt{23}\,v)=(1+i\sqrt{23})(a+i\sqrt{23}\,b)^2 \tag{6}$$

Equating real and imaginary parts, we have

$$u = a^2 - 23b^2 - 46ab - 1$$
$$v = a^2 - 23b^2 + 2ab$$

Substituting the above values of u and v in (2), the values of *x* and y are given by

$$\left.\begin{aligned} x &= x(a, b) = 2a^2 - 46b^2 - 44ab - 1 \\ y &= y(a, b) = -48ab - 1 \end{aligned}\right\} \quad (7)$$

Thus (5) and (7) represent non zero distinct integral solutions of (1) in two parameters.

Properties

1. $x(a,1) + 2z(a,1) - t_{10,a} \equiv 40 \pmod{41}$
2. $-6 \times 41\left(x(\alpha^2,1) - 2z(\alpha^2,1) - t_{10,\alpha^2} + 1\right)$ is a nasty number.
3. $x(a,1) + 2z(a,1) - 8t_{3,a-1} \equiv 1 \pmod{40}$
4. $2y(a,1) + 2z(a,1) - t_{6,a} \equiv 44 \pmod{95}$
5. $-t_{10,a} + 8t_{3,a-1} = -a$
6. $x(a,1) + 2y(a,1) + 4z(a,1) - t_{10,a} - t_{6,a} = -136a + 43$

Pattern: 2

Consider (3) as

$$u^2 - z^2 = 23(z^2 - v^2) \quad (8)$$

where $u + 1 = U$

Write (8) in the form of ratio as

$$\frac{u + z}{23(z + v)} = \frac{z - v}{u - z} = \frac{\alpha}{\beta}, \quad \beta \neq 0$$

which is equivalent to the following two equations

$$\beta u - 23\alpha v + z(\beta - 23\alpha) = 0$$
$$-\alpha u - \beta v + z(\alpha + \beta) = 0$$

On employing the method of cross multiplication we get

$$\left.\begin{aligned} u &= -23\alpha^2 + \beta^2 - 46\alpha\beta - 1 \\ v &= 23\alpha^2 - \beta^2 - 2\alpha\beta \end{aligned}\right\} \quad (9)$$

$$z = -\beta^2 - 23\alpha^2 \quad (10)$$

Substituting the values of u and v in (2), the non - zero distinct integral values of *x* and y are given by

$$\left.\begin{aligned} x &= x(\alpha, \beta) = -48\alpha\beta - 1 \\ y &= y(\alpha, \beta) = -46\alpha^2 + 2\beta^2 - 44\alpha\beta - 1 \end{aligned}\right\} \quad (11)$$

Thus (10) and (11) represent the nonzero distinct integer solutions of (1) in two parameters.

Properties

- $y(\alpha,1) + 2z(\alpha,-1) + t_{186,\alpha} \equiv 134 \pmod{135}$

- $-6\times 135\ [y(k^2,1)+2z(k^2,1)+t_{186,k^2}+1]$ is a nasty number
- $y(\alpha,1)+2z(\alpha,1)+184\ t_{3,\alpha-1}\equiv 135 \pmod{136}$
- $-6\times 136\left[y(k^2,1)+2z(k^2+1)+184\ t_{3,k^2-1}+1\right]$
- $x(\alpha,1)+y(\alpha,1)+3z(\alpha,1)+46t_{3,\alpha}+t_{186,\alpha}=-160\alpha-3$
- $t_{186,\alpha}-184\ t_{3,\alpha-1}=\alpha$
- $2y(\alpha,1)+4z(\alpha,1)+t_{186,\alpha}+184\ t_{3,\alpha-1}=-271\alpha-2$

Pattern: 3
Write (8) in the form of ratio as

$$\frac{U+z}{z+v}=\frac{23(z-v)}{U-z}=\frac{\alpha}{\beta}\quad,\ \beta\neq 0$$

which is equivalent to the following two equations

$$\beta U-\alpha v+z(\beta-\alpha)=0$$
$$-\alpha U-23\beta v+z(\alpha+23\beta)=0$$

On employing the method of cross multiplication, we get

$$u=-\alpha^2+23\beta^2-46\alpha\beta-1 \text{ where } u=U-1$$
$$v=\alpha^2-23\beta^2-2\alpha\beta$$
$$z=-\alpha^2-23\beta^2$$

Substituting the values of u and v in (2), the non - zero distinct integral values of x and y are given by,

$$\left.\begin{aligned} x&=x(\alpha,\beta)=-48\alpha\beta-1\\ y&=y(\alpha,\beta)=-2\alpha^2+46\beta^2-44\alpha\beta-1\end{aligned}\right\}\qquad(12)$$

Thus, (11) and (12) represent the non - zero distinct integer solutions of (1) in two parameters.

Properties

- $y(\alpha,1)+2z(\alpha,1)+t_{10,\alpha}\equiv 46 \pmod{47}$
- $y(\alpha,1)+2z(\alpha,1)+8t_{3,\alpha}\equiv 39 \pmod{40}$
- $-6\times 47\left[y(k^2,1)+2z(k^2+1)+t_{10,k^2}+1\right]$ is a nasty number
- $7x(\alpha,1)+y(\alpha,1)+9z(\alpha,1)+t_{10,\alpha}+t_{16,\alpha}=-389\alpha-169$
- $2y(\alpha,1)+4z(\alpha,1)+t_{10,\alpha}+8t_{3,\alpha}=-87\alpha-2$
- $t_{10,\alpha}-8t_{3,\alpha}=-7\alpha$

Pattern: 4
Write (8) in the form of ratio as

$$\frac{U-z}{23(z-v)}=\frac{z+v}{U+z}=\frac{\alpha}{\beta}\quad,\ \beta\neq 0$$

which is equivalent to the following two equations

$$\beta U+23\alpha v-z(\beta+23\alpha)=0$$
$$-\alpha U+\beta v+z(\beta-\alpha)=0$$

On employing the method of cross multiplication we get

$$u = -\alpha^2 + \beta^2 + 24\alpha\beta - 1 \text{ where u=U-1}$$
$$v = 23\alpha^2 - \beta^2 + 2\alpha\beta$$
$$z = 23\alpha^2 + \beta^2 \quad (13)$$

Substituting the values of u and v in (2), the non - zero distinct integral values of x and y are given by,

$$\left.\begin{array}{l} x = x(\alpha,\beta) = 48\alpha\beta - 1 \\ y = y(\alpha,\beta) = -2\alpha^2 + 46\beta^2 + 44\alpha\beta - 1 \end{array}\right\} \quad (14)$$

Thus, (13) and (14) represent the non - zero distinct integer solutions of (1) in two parameters.

Properties

- $y(\alpha,1) - 2z(\alpha,1) + t_{142,\alpha} \equiv 46 \pmod{47}$
- $6 \times 71\left[x(k^2,1) + z(k^2+1) - 90\, t_{3,k^2-1} + 1\right]$ is a nasty number
- $x(\alpha,1) + y(\alpha,1) - z(\alpha,1) + t_{142,\alpha} - 90\, t_{3,\alpha-1} = 24\alpha - 1$
- $2y(\alpha,1) - 4z(\alpha,1) + t_{142,\alpha} + t_{52,\alpha} + t_{42,\alpha} + t_{32,\alpha} + t_{22,\alpha} = -91\alpha - 2$
- $t_{142,\alpha} + t_{52,\alpha} + t_{42,\alpha} + t_{32,\alpha} + t_{22,\alpha} = -3\alpha$

Pattern: 5
Write (8) in the form of ratio as

$$\frac{U - z}{z - v} = \frac{23(z+v)}{U+z} = \frac{\alpha}{\beta}, \quad \beta \neq 0$$

which is equivalent to the following two equations

$$\beta U + \alpha v - z(\beta + \alpha) = 0$$
$$-\alpha U + 23\beta v + z(23\beta - \alpha) = 0$$

On employing the method of cross multiplication we get

$$u = -\alpha^2 - 23\beta^2 + 46\alpha\beta - 1 \text{ where u=U-1}$$
$$v = \alpha^2 - 23\beta^2 + 2\alpha\beta$$
$$z = \alpha^2 + 23\beta^2 \quad (15)$$

Substituting the values of u and v in (2), the non - zero distinct integral values of x and y are given by,

$$\left.\begin{array}{l} x = x(\alpha,\beta) = 48\alpha\beta - 1 \\ y = y(\alpha,\beta) = -2\alpha^2 + 46\beta^2 + 44\alpha\beta - 1 \end{array}\right\} \quad (16)$$

Thus (15) and (16) represent the non - zero distinct integer solutions of (1) in two parameters.

Properties

- $y(1,\beta) + 2z(1,\beta) - t_{186,\beta} \equiv 134 \pmod{135}$
- $x(1,\beta) + y(1,\beta) + 3z(1,\beta) - 46\, t_{3,\beta} - t_{186,\beta} \equiv 160\beta - 1$
- $6 \times 135\,[y(1,\beta) + 2z(1,\beta) - t_{186,\beta} + 1]$ is a nasty number

- $t_{20,\beta} + t_{8,\beta} + t_{28,\beta} + t_{24,\beta} + t_{36,\beta} + t_{12,\beta} + t_{46,\beta} + t_{26,\beta} - t_{186,\beta} = 7\beta$

  $2y(1,\beta) + 4z(1,\beta) - t_{186,\beta} - t_{20,\beta} - t_{8,\beta} - t_{28,\beta} - t_{24,\beta} - t_{36,\beta} - t_{12,\beta} - t_{46,\beta} - t_{26,\beta} = 263\beta - 2$

Pattern: 6

Consider (3) as

$$(u+1)^2 + 23v^2 = 24z^2 \times 1$$

Let $U = u + 1$

$$\therefore \quad U^2 + 23v^2 = 24z^2 \times 1 \tag{17}$$

Consider,

$$U^2 + 23v^2 = \left(U + i\sqrt{23}\,v\right)\left(U - i\sqrt{23}\,v\right) \tag{18}$$

$$24 = \left(1 + i\sqrt{23}\right)\left(1 - i\sqrt{23}\right) \tag{19}$$

Write (5) as

$$z^2 = \left(a + i\sqrt{23}\,b\right)^2\left(a - i\sqrt{23}\,b\right)^2 \tag{20}$$

Write 1 as

$$1 = \frac{\left(11 + i\sqrt{23}\right)\left(11 - i\sqrt{23}\right)}{12^2} \tag{21}$$

Using (18), (19), (20), (21) in (17) and employing the method of factorization, define,

$$\left(U + i\sqrt{23}\,v\right) = \left(1 + i\sqrt{23}\right)\left(a + i\sqrt{23}\,b\right)^2 \times \frac{\left(11 + i\sqrt{23}\right)}{12}$$

Equating real and imaginary parts, we have

$$u = \left[-a^2 + 23b^2 - 46ab - 1\right]$$

$$v = \left[a^2 - 23b^2 - 2ab\right]$$

$$z = z(a,b) = a^2 + 23b^2$$

Substituting u and v in (4.2) we have

$$\left.\begin{aligned} x &= x(a,b) = -48ab - 1 \\ y &= y(a,b) = -2a^2 + 46b^2 - 44ab - 1 \end{aligned}\right\} \tag{22}$$

Thus, (5) and (22) represent the non - zero distinct integer solutions of (1) in two parameters.

Properties

- $y(1,b) + 2z(1,b) - t_{186,b} \equiv 46 \pmod{47}$
- $x(1,b) + y(1,b) + 3z(1,b) - t_{22,b} - t_{186,b} - t_{14,b} - t_{16,b} \equiv 19b - 1$
- $6 \times 47\,[y(1,b) + 2z(1,b) - t_{186,b} + 1]$ is a nasty number
- $184\,t_{3,b} - t_{186,b} \equiv 183b$
- $2y(1,b) + 4z(1,b) - 184\,t_{3,b} - t_{186,b} = 183b - 2$

Pattern: 7
Write (3) as

$$U^2 = 24z^2 - 23v^2 \text{ where } U = u + 1 \tag{23}$$

Introducing the linear transformations

$$\left.\begin{aligned} z &= X + 23T \\ v &= X + 24T \end{aligned}\right\} \tag{24}$$

Substituting the values of z, v and u in (23) we get,

$$X^2 = U^2 + 552T^2 \tag{25}$$

which is satisfied by

$$\left.\begin{aligned} U &= 552p^2 - q^2 \\ X &= 552p^2 + q^2 \\ T &= 2pq \end{aligned}\right\} \tag{26}$$

Substituting the values of U, X, and T in (24) we get,

$$\left.\begin{aligned} u &= 552p^2 - q^2 - 1 \\ v &= 552\ p^2 + q^2 + 48pq \\ z &= 552p^2 + q^2 + 46pq \end{aligned}\right\} \tag{27}$$

Substituting (24) in (2), we get

$$\left.\begin{aligned} x &= x(p, q) = 1104p^2 + 48pq\ -1 \\ y &= y(p, q) = -2q^2 - 48pq\ -1 \\ z &= 552p^2 + q^2 + 46pq \end{aligned}\right\} \tag{28}$$

Thus, (29) represent the non zero distinct integer solutions of (1) in two parameters.

Properties

- $y(p,1) + 2z(p,1) - t_{2206,p} \equiv 1146 \pmod{1147}$
- $2y(p,1) + 4z(p,1) - 2208\ t_{3,p} - t_{2206,p} \equiv 87p\ - 2$
- $-6 \times 1060\left[y(k^2 - 1) + 2z(k^2,1) - 2208\ t_{3,k^2-1} + 1\right]$ is a Nasty number
- $t_{2206,p} - 2208\ t_{3,p} - t_{2206,p} = -2207p$

Note
Instead of (24), one may also consider the transformations as

$$\left.\begin{aligned} z &= X - 23T \\ v &= X - 24T \end{aligned}\right\} \tag{29}$$

For the choice, the corresponding integer solutions are

$$\left.\begin{aligned} x &= x(p,q) = 1104p^2 - 48pq - 1 \\ y &= y(p,q) = -2q^2 + 48pq - 1 \\ z &= 552p^2 + q^2 - 46pq \end{aligned}\right\} \quad (30)$$

Thus (30) represent the non zero distinct integral solutions of (1) in two parameters.

Pattern: 8

It is worth to note that (25) is satisfied by other choices of X, T, U which is illustrated as below (25) is written as the systems of two equations as follows:

Table 1

SYSTEM	X+U	X-U
1	T^2	552
2	$2T^2$	276
3	$4T^2$	138
4	$6T^2$	92
5	$8T^2$	69
6	$12T^2$	46
7	$276T^2$	2
8	$138T^2$	4
9	$92T^2$	6
10	$69T^2$	8
11	$46T^2$	12
12	552T	T
13	46T	2T
14	69T	8T
15	92T	6T
16	138T	4T
17	276T	2T

System: 1

$$X + U = T^2 ; X - U = 552$$

And solving, we get

$$X = 2k^2 + 276 ; U = 2k^2 - 276 ; T = 2k$$

Substituting the above values of X, T, U in (24), and employing (2), the corresponding integer solutions to (1) are as follows

$$\left.\begin{aligned} x &= x(k) = 4k^2 + 48k - 1 \\ y &= y(k) = -48k - 553 \\ z &= z(k) = 2k^2 + 276k + 46 \end{aligned}\right\} \quad (31)$$

Thus, (31) represent the non zero distinct integer solutions of (1) in two parameters.
For simplicity, the values of x, y, z satisfying (1) obtained on solving the systems 2 to 17 are presented below.

Table 2

System: 2 $x = x(T) = 2T^2 + 24T - 1$ $y = y(T) = -24T - 277$ $z = z(T) = T^2 + 23T + 138$	**System: 10** $x = x(T) = 276K^2 + 48K - 1$ $y = y(T) = -48K - 9$ $z = z(T) = 138K^2 + 46K + 4$
System: 3 $x = x(T) = 4T^2 + 24T - 1$ $y = y(T) = -24T - 139$ $z = z(T) = 2T^2 + 23T + 69$	**System: 11** $x = x(T) = 46T^2 + 24T - 1$ $y = y(T) = -24T - 13$ $z = z(T) = 23T^2 + 23T + 6$
System: 4 $x = x(T) = 6T^2 + 24T - 1$ $y = y(T) = -24T - 93$ $z = z(T) = 3T^2 + 23T + 46$	**System:12** $x = x(T) = 1152K - 1$ $y = y(T) = -50K - 1$ $z = z(T) = 599K$
System: 5 $x = x(T) = 32K^2 + 80K + 92$ $y = y(T) = -48K - 26$ $z = z(T) = 16K^2 + 62K + 58$	**System: 13** $x = x(T) = 70T - 1$ $y = y(T) = -26T - 1$ $z = z(T) = 47T$
System: 6 $x = x(T) = 12T^2 + 24T - 1$ $y = y(T) = -24T - 47$ $z = z(T) = 6T^2 + 23T + 23$	**System: 14** $x = x(T) = 186K - 1$ $y = y(T) = -64K - 1$ $z = z(T) = 123K$
System: 7 $x = x(T) = 276T^2 + 24T - 1$ $y = y(T) = -24T - 3$ $z = z(T) = 138T^2 + 23T + 1$	**System: 15** $x = x(T) = 232K - 1$ $y = y(T) = -60K - 1$ $z = z(T) = 144K$
System: 8 $x = x(T) = 138T^2 + 24T - 1$ $y = y(T) = -24T - 5$ $z = z(T) = 69T^2 + 23T + 2$	**System: 16** $x = x(T) = 162T - 1$ $y = y(T) = -28T - 1$ $z = z(T) = 94T$
System: 9 $x = x(T) = 92T^2 + 24T - 1$ $y = y(T) = -24T - 7$ $z = z(T) = 46T^2 + 23T + 3$	**System: 17** $x = x(T) = 300T - 1$ $y = y(T) = -26T - 1$ $z = z(T) = 162T$

3. Conclusion

In this paper, we have presented different patterns of integer solutions to the ternary quadratic equation $6(x^2 + y^2) - 11xy + x + y + 1 = 24z^2$. As this Diophantine equations are rich in variety, one may attempt to find integer solutions to other choices of equations along with suitable properties.

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